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BarcelonaTECH, Spain

Lecture 7

Code pseudorange modelling

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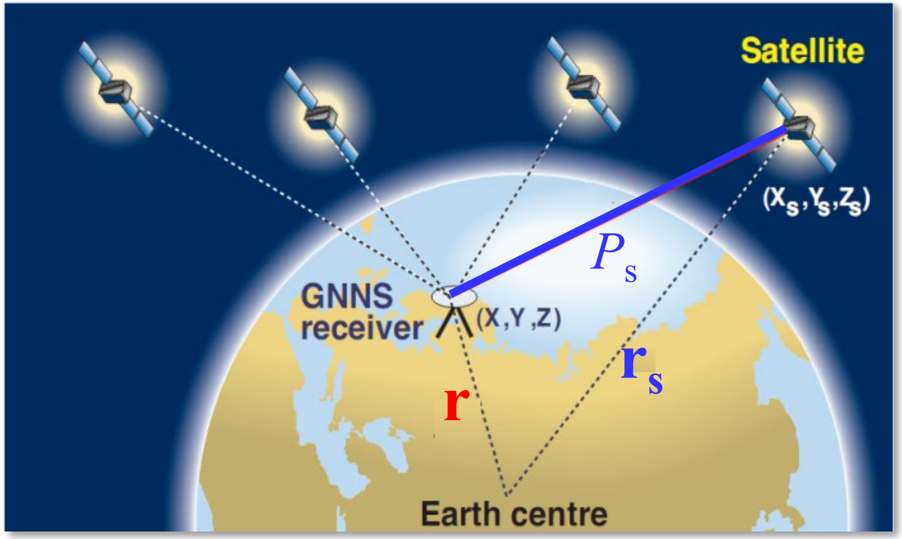
Introduction: Linear model and Prefit-residuals



Input:

- **Pseudoranges (receiver-satellite j):** P_s
- **Navigation message. In particular:**
  - **Satellites position when transmitting signal:** $\mathbf{r}_s = (x_s, y_s, z_s)$
  - **Offsets of satellite clocks:** dt_s

  (satellites = 1, 2,…n) (n>=4)

Unknowns:

- **Receiver position:** $\mathbf{r} = (x, y, z)$
- **Receiver clock offset:** dT

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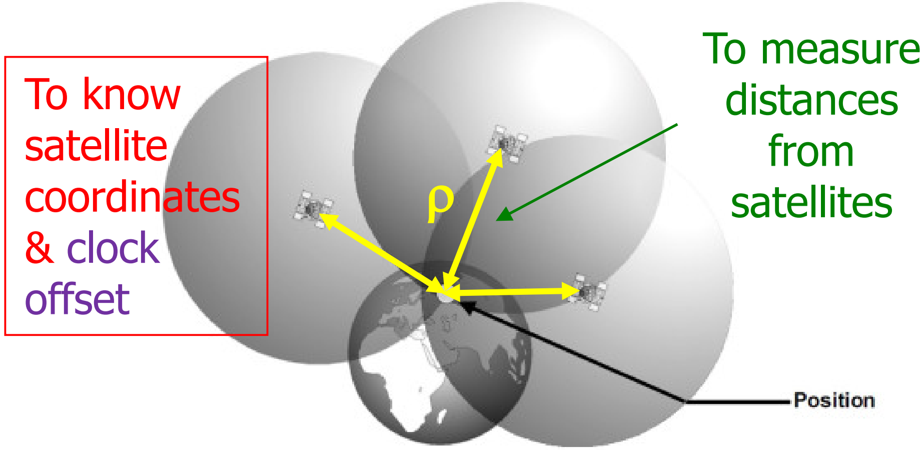
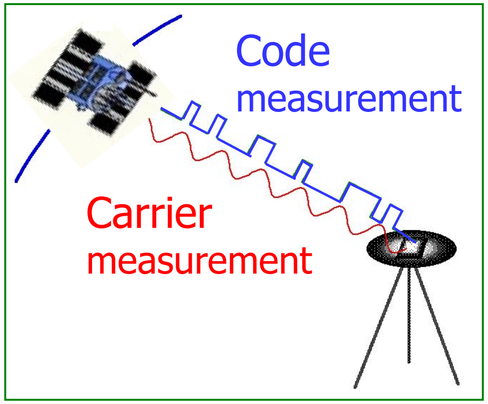
GNSS positioning concept





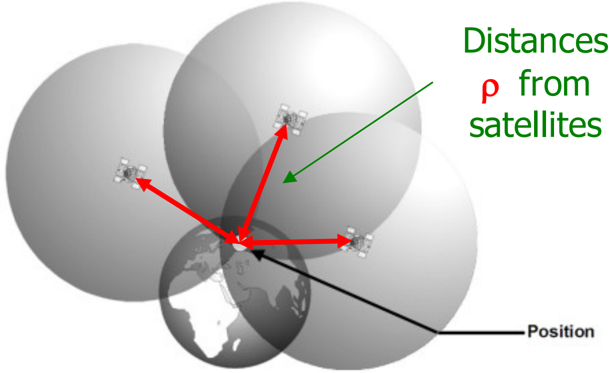
This picture is from https://gpsfleettrackingexpert.wordpress.com

- GNSS uses technique of "**triangulation**" to find user location
- To "**triangulate**" a GNSS receiver needs:
  - **To know the satellite coordinates** and clock synchronism errors:
    ➔ Satellites broadcast orbits parameters and clock offsets.
  - **To measure distances from satellites**:
    ➔ This is done measuring the **traveling time** of radio signals: ("Pseudo-ranges": **Code** and **Carrier** measurements)

    ➔ Measurements must be corrected by several error sources: Atmospheric propagation, relativity, clock offsets, instrumental delays…

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_{1rec} + TGD^{sat} + \varepsilon_1$$

Figure 6.1: Geometric concept of GNSS positioning: Equations



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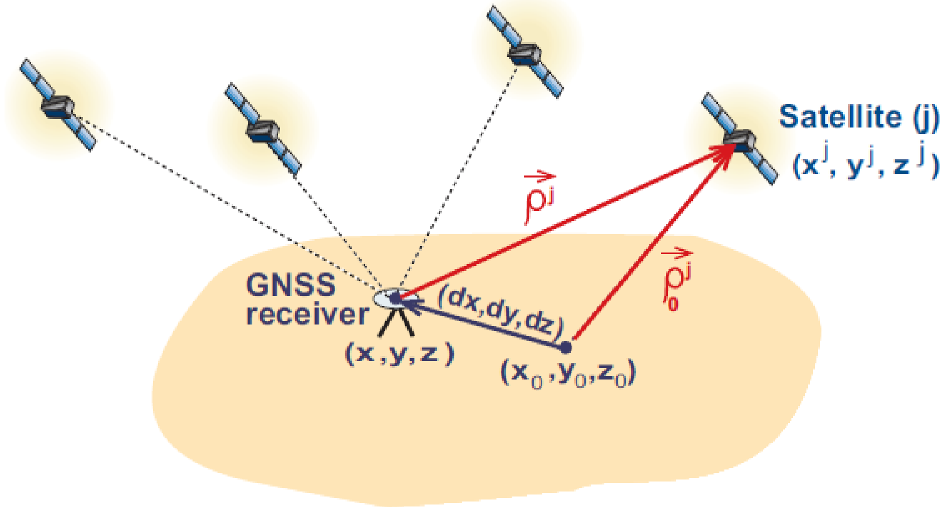

Then, linearising the satellite–receiver geometric range

$$\rho^j(x,y,z) = \sqrt{(x^j - x)^2 + (y^j - y)^2 + (z^j - z)^2}$$

gives, for the approximate solution $\mathbf{r}_0 = (x_0, y_0, z_0)$,

$$\rho^j = \rho_0^j + \frac{x_0 - x^j}{\rho_0^j} dx + \frac{y_0 - y^j}{\rho_0^j} dy + \frac{z_0 - z^j}{\rho_0^j} dz$$

with $dx = x - x_0, \; dy = y - y_0, \; dz = z - z_0$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(d\bar{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

For each satellite in view

Iono+Tropo+TGD…

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + \sum \delta_k + \varepsilon$$

Linearising ρ around an 'a priori' receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$

$$= \rho_{0,rec}^{sat} + \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}}\Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}}\Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}}\Delta z_{rec} + c\left(dt_{rec} - dt^{sat}\right) + \sum \delta_k$$

where:

$$\Delta x_{rec} = x_{rec} - x_{0,rec} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{0,rec} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{0,rec}$$

Prefit-residuals (Prefit)

$$C1_{rec}^{sat} - \rho_{0,rec}^{sat} + c\,dt^{sat} - \sum \delta_k = \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}}\Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}}\Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}}\Delta z_{rec} + c\,dt_{rec}$$

measurement | computed | **unknown**

$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2}$$

Of course, receiver coordinates $(x_{rec}, y_{rec}, z_{rec})$ are not known (they are the target of this problem). But, we can always assume that an "approximate position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$ is known".

Thence, the navigation problem will consist on:

1.- To start from an approximate value for receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$ e.g. the Earth's centre) to linearise the equations.

2.- With the pseudorange measurements and the navigation equations, compute the correction $(\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})$ to have improved estimates: $(x_{rec}, y_{rec}, z_{rec}) = (x_{0,rec}, y_{0,rec}, z_{0,rec}) + (\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})$

3.- Linearise the equations again, about the new improved estimates, and iterate until the change in the solution estimates is sufficiently small.

The estimates converges quickly. Generally in two to four iterations, even if starting from the Earth's Centre.

For each satellite in view

Iono+Tropo+TGD...

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + \sum \delta_k + \varepsilon$$

Linearising ρ around an 'a priori' receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$

$$= \rho_{0,rec}^{sat} + \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}} \Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}} \Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}} \Delta z_{rec} + c\left(dt_{rec} - dt^{sat}\right) + \sum \delta_k$$

where:

$$\Delta x_{rec} = x_{rec} - x_{0,rec} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{0,rec} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{0,rec}$$

Prefit-residuals (Prefit)

$$C1_{rec}^{sat} - \rho_{0,rec}^{sat} + c\,dt^{sat} - \sum \delta_k = \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}} \Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}} \Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}} \Delta z_{rec} + c\,dt_{rec}$$

measurement | computed | unknown

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For all satellites in view

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \cdots\cdots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{0,rec} - x^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{y_{0,rec} - y^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{z_{0,rec} - z^{sat1}}{\rho_{0,rec}^{sat1}} & 1 \\ \frac{x_{0,rec} - x^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{y_{0,rec} - y^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{z_{0,rec} - z^{sat2}}{\rho_{0,rec}^{sat2}} & 1 \\ & \cdots\cdots\cdots & & \\ \frac{x_{0,rec} - x^{sat\,n}}{\rho_{0,rec}^{sat\,n}} & \frac{y_{0,rec} - y^{sat\,n}}{\rho_{0,rec}^{sat\,n}} & \frac{z_{0,rec} - z^{sat\,n}}{\rho_{0,rec}^{sat\,n}} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c\,dt_{rec} \end{bmatrix}$$

Observations
(measured-modelled)

Unknowns

Measurements modelling:

Prefit residual is the difference between measured and modeled pseudorange:

$$\text{Prefit}_{rec}^{sat} = C1_{rec}^{sat}[\text{measured}] - C1_{rec}^{sat}[\text{modelled}]$$

where:

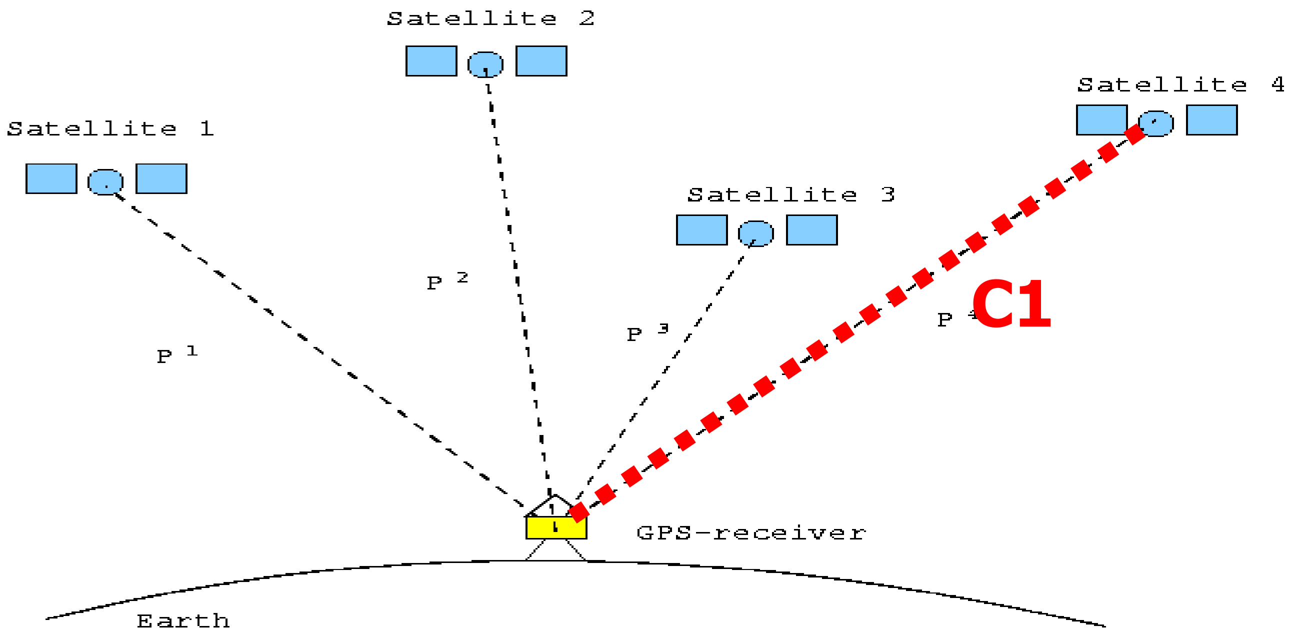
$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(\overline{dt}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

dt^{sat}

Contents

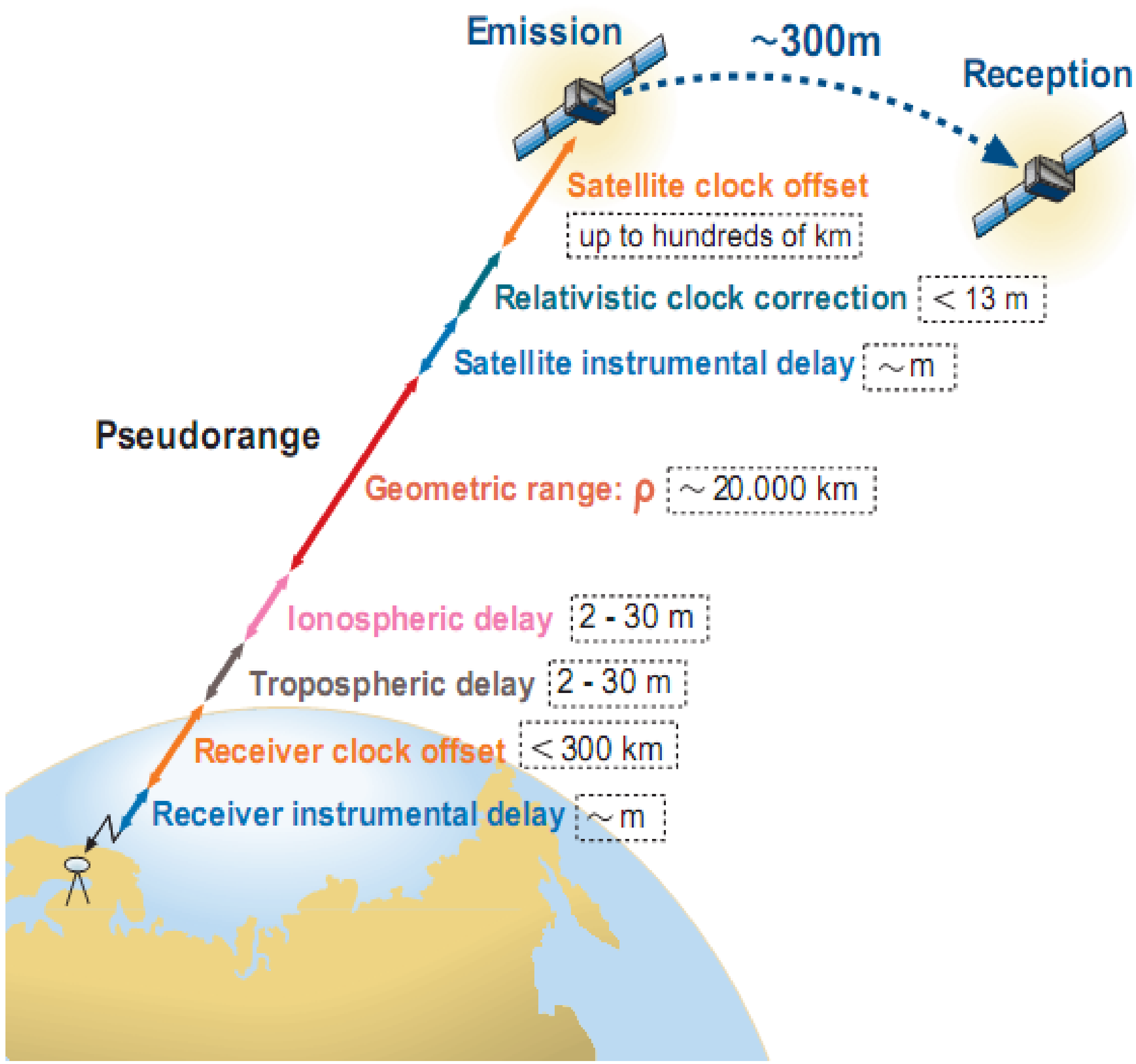
Measurements modelling and error sources

Code Pseudorange modeling



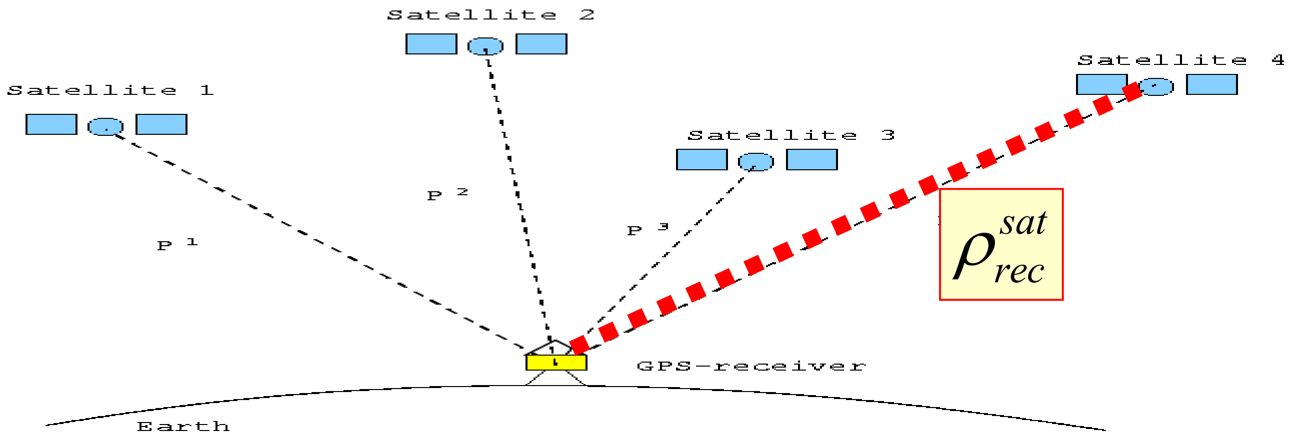
The pseudorange modeling is based in the GPS Standard Positioning Service Signal Specification (GPS/SPS-SS).

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(\overline{dt}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{\,sat} + TGD^{sat}$$



$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(d\bar{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Geometric range



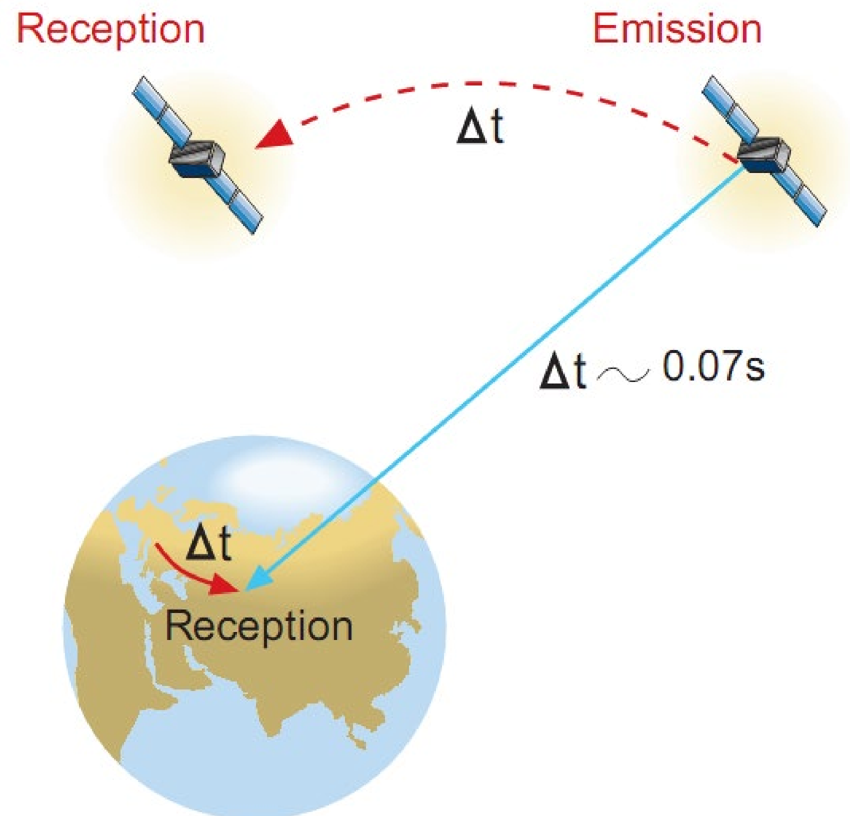
Euclidean distance between satellite coordinates at emission time and receiver coordinates at reception time.

$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2}$$

Of course, receiver coordinates are not known (is the target of this problem). But ....

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} + c\left(\overline{dt}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Satellite coordinates **at emission time** (rec2ems.f)



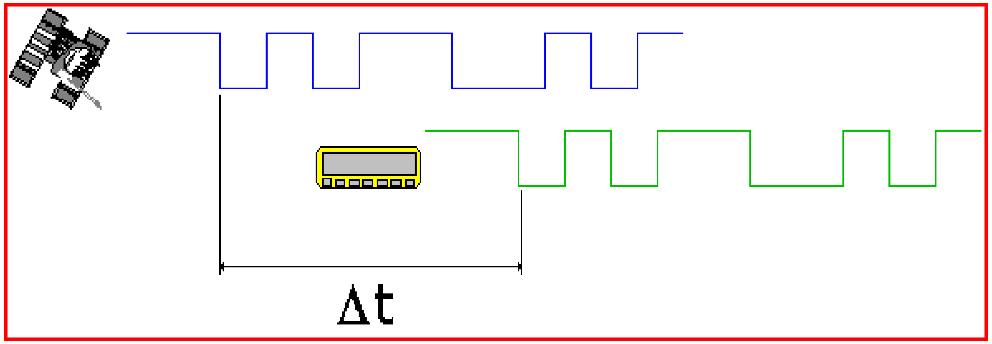
- The GPS signal travels from satellite coordinates at emission time (T_{emis}) to receiver coordinates at reception time (T_{recep}).

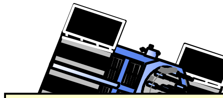
- The satellite can move several hundreds of meters from T_{emis} to T_{recep}.

The receiver time-tags are given at reception time and in the receiver clock time.

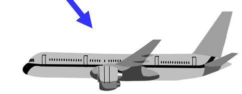
An algorithm is needed to compute the satellite coordinates at **emission time** "in the GPS system time" from reception time in the receiver time tags.

Emission time in the GPS system time T[ems]





The satellite clock offset dt^S can be computed from the navigation message



$$C1 = c\,\Delta t = c\,[t_R(T_{recep}) - t^S(T_{emis})]$$

As it is known, the pseudorange measurements link the "emission time (T_{emis})" in satellite clock (t^S) with reception time (T_{recep}) in receiver clock (t_R) (receiver time tags).

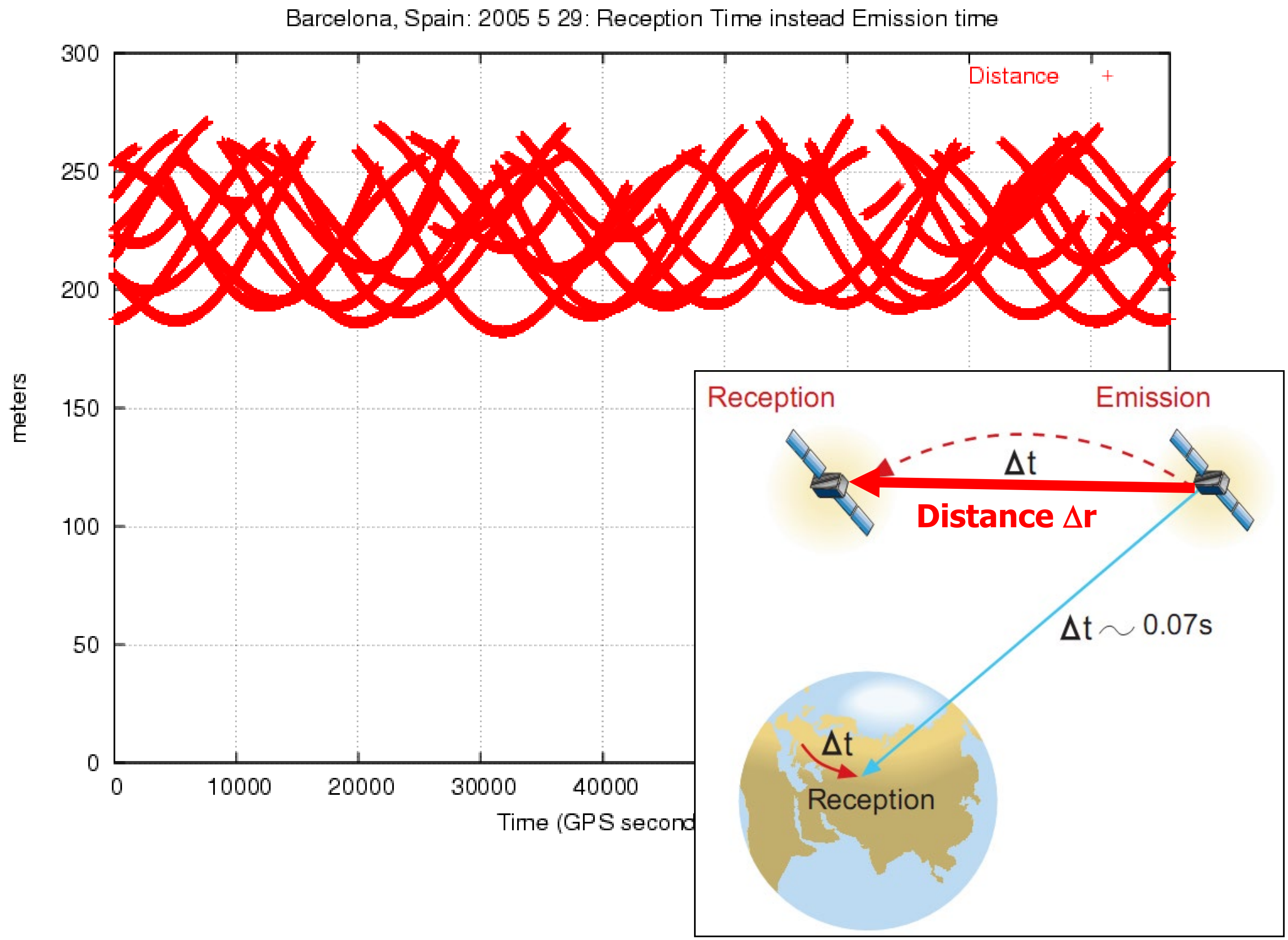
Thence, the emission time in the satellite clock is:

$$t^S(T_{emis}) = t_R(T_{recep}) - C1/c$$

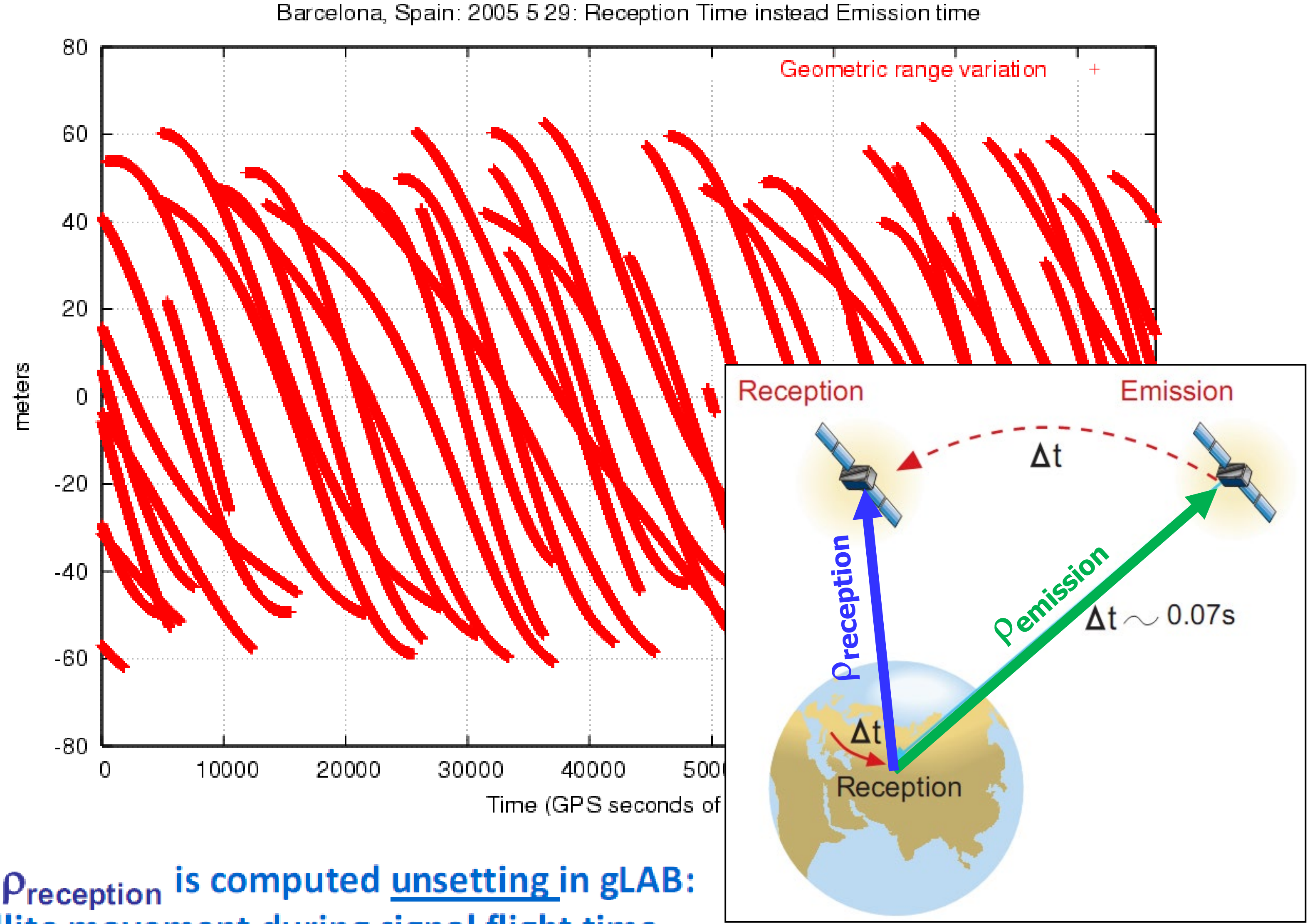
Finally, since $dt^S = t^S - T$ is the time offset between satellite clock (t^S) and **GPS system time** (T), thence:

$$T_{emis} = t^S(T_{emis}) - dt^S = t_R(T_{recep}) - C1/c - dt^S$$

Distance: Δr



Variation in range: $\Delta\rho = \rho_{emission} - \rho_{reception}$



Note: $\rho_{reception}$ is computed unsetting in gLAB:
- **Satellite movement during signal flight time.**
- **Earth rotation during signal flight time.**

@ J. Sanz & J.M. Juan

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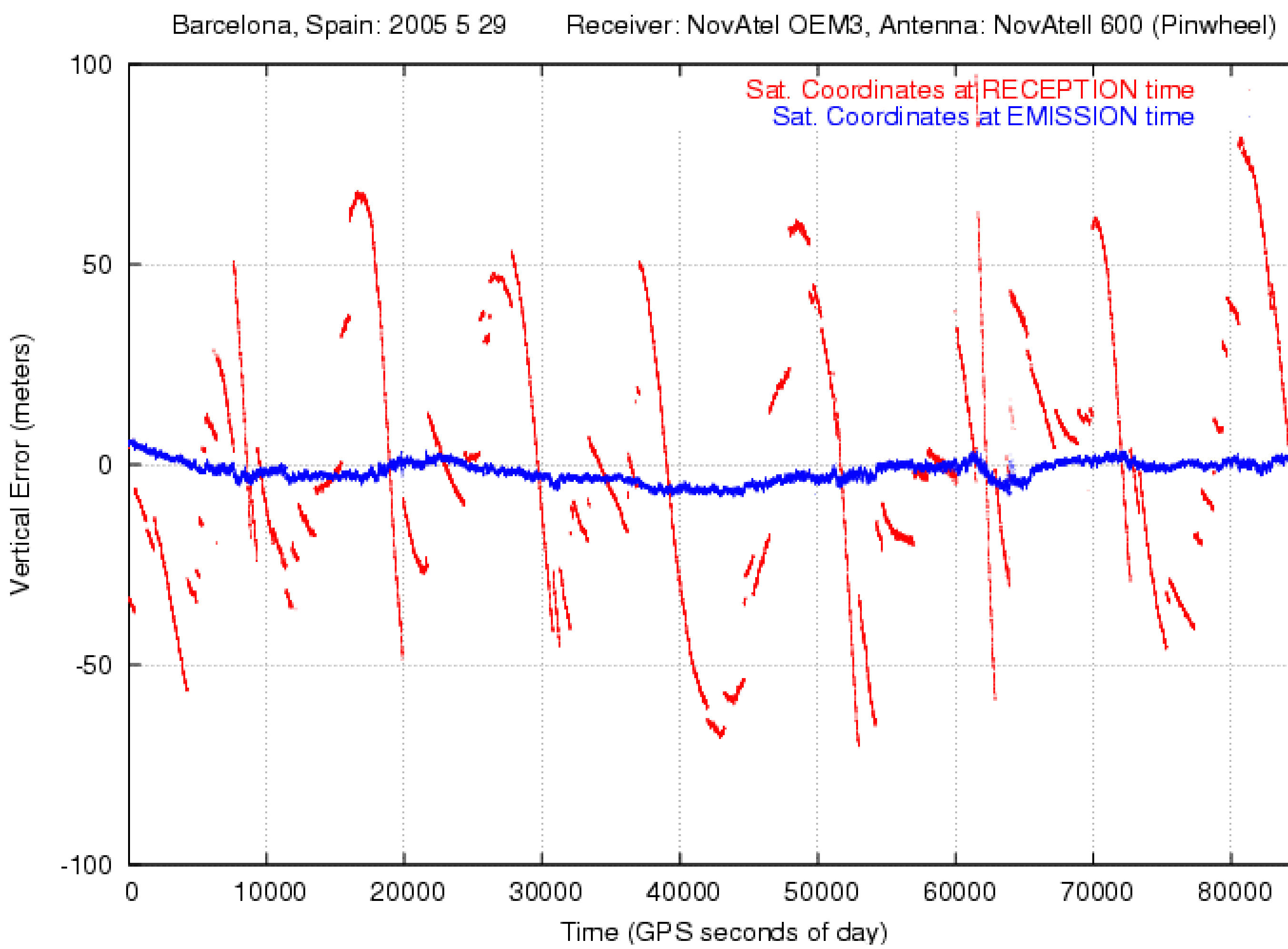
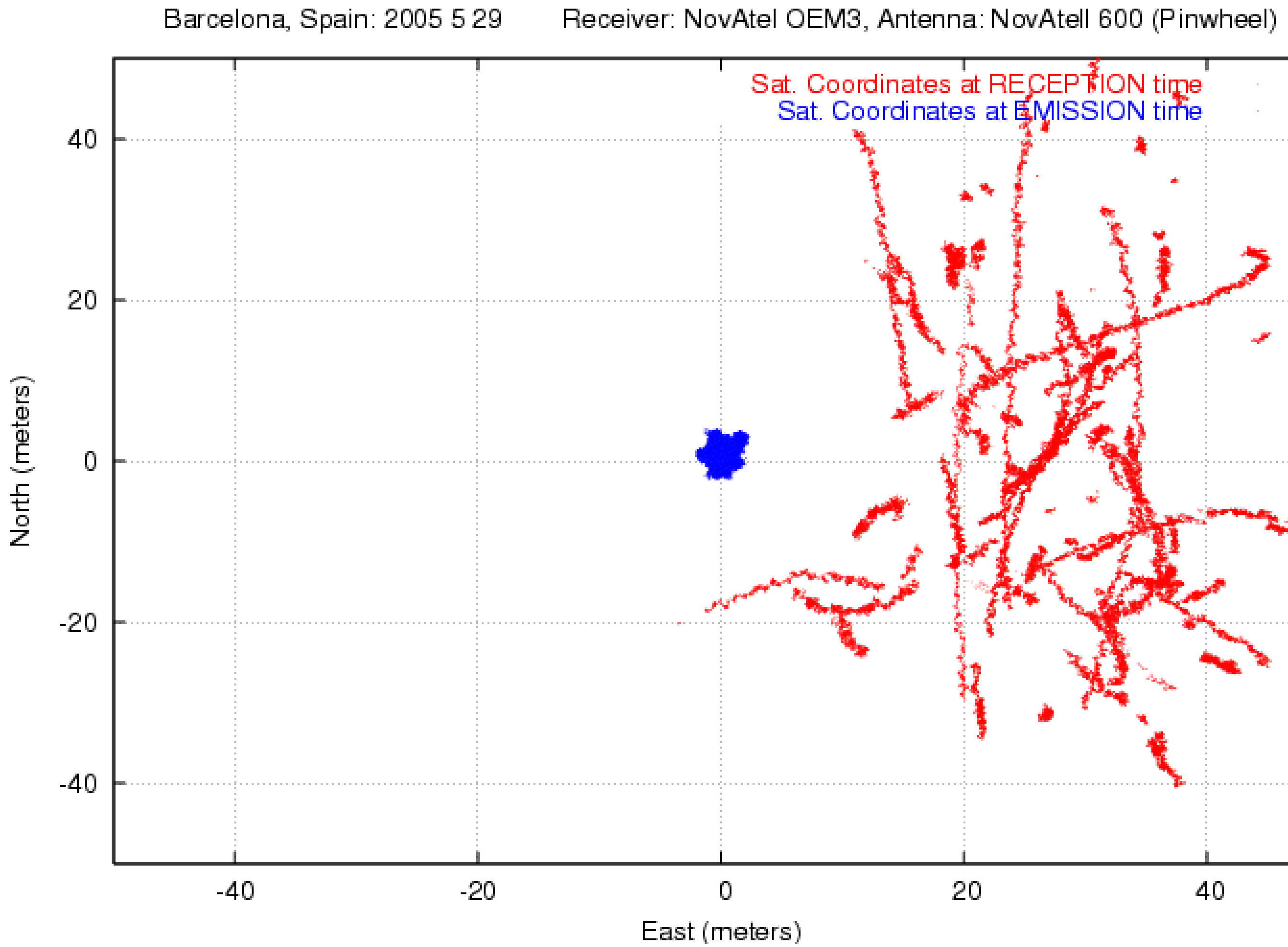
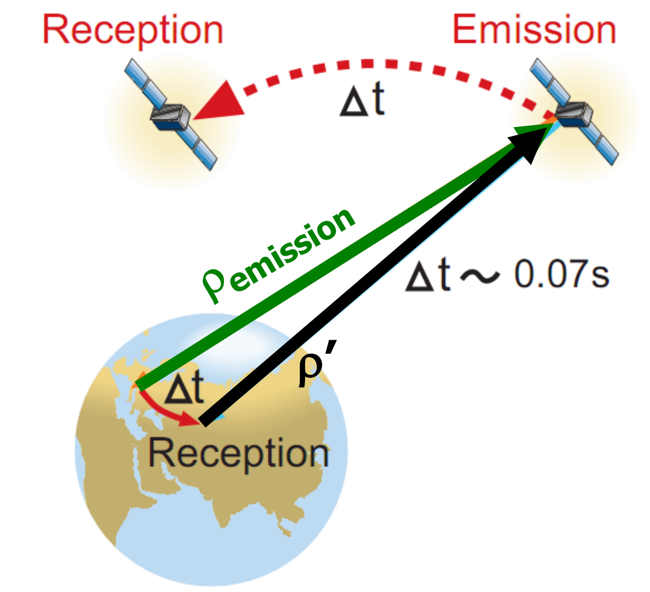
Vertical error comparison



Horizontal error comparison





...rdinates computation **at emission time**

...vided by the GPS/SPS-SS (**orbit.f**) supplies satellite ... an Earth-Fixed reference frame. To compute the ... nates

See **rec2ems.f**

...ne, the following algorithm can be applied:

... time-tags, compute emission time in GPS system time:

$$T_{emis} = t_R(T_{recep}) - (C1/c + dt^S)$$

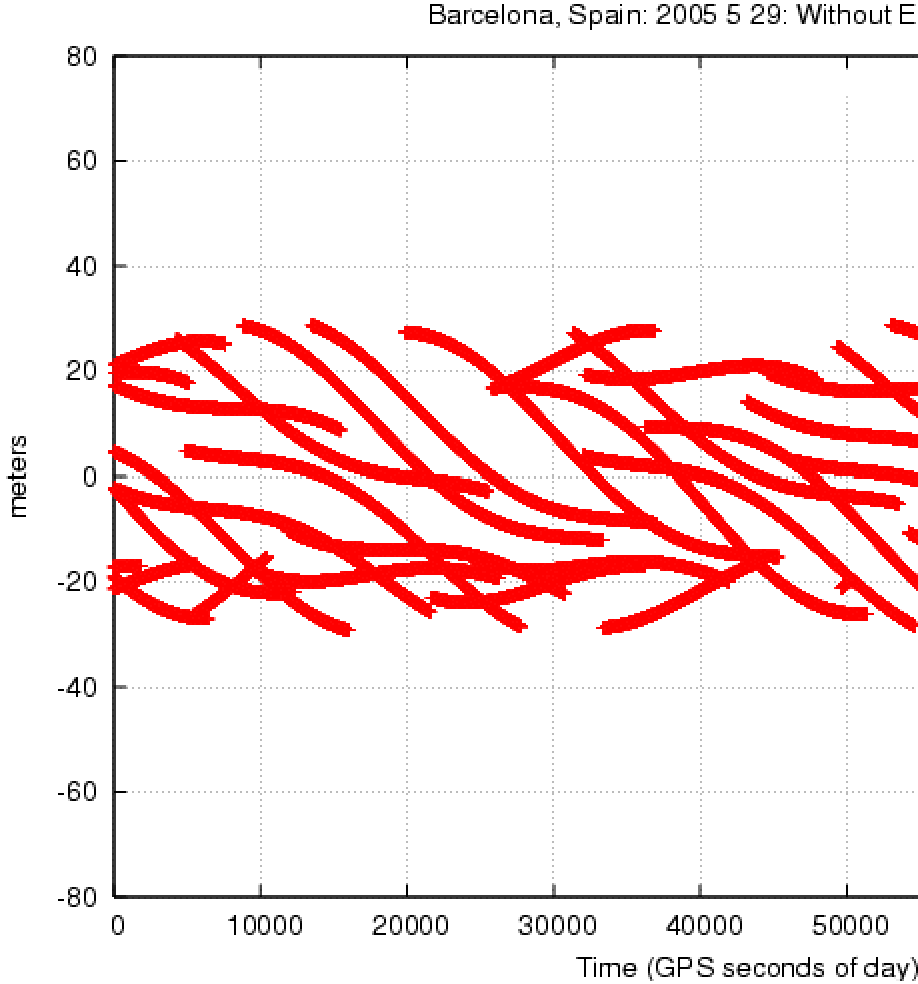
2. Compute satellite coordinates at emission time T_{emis}

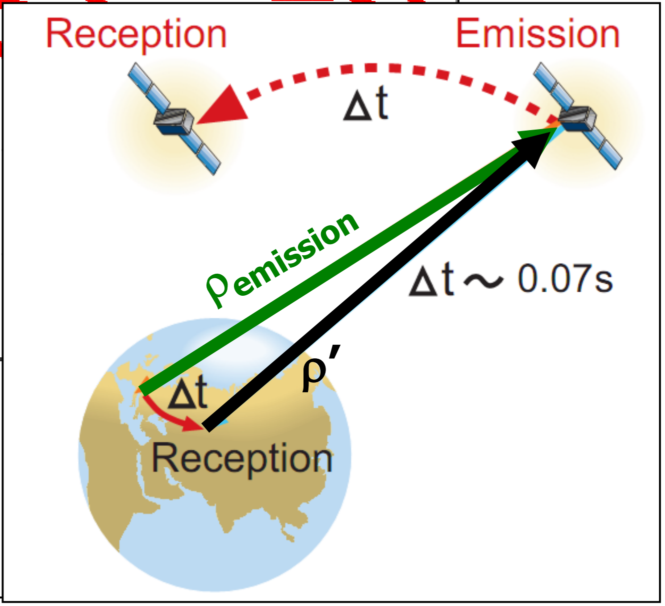
$$T_{emis} \rightarrow \textbf{[orbit]} \rightarrow (X^{sat}, Y^{sat}, Z^{sat})_{CTS[emission]}$$

3. Account for Earth rotation during traveling time from emission to reception "Δt" *(CTS reference system at reception time is used to build the navigation equations).*

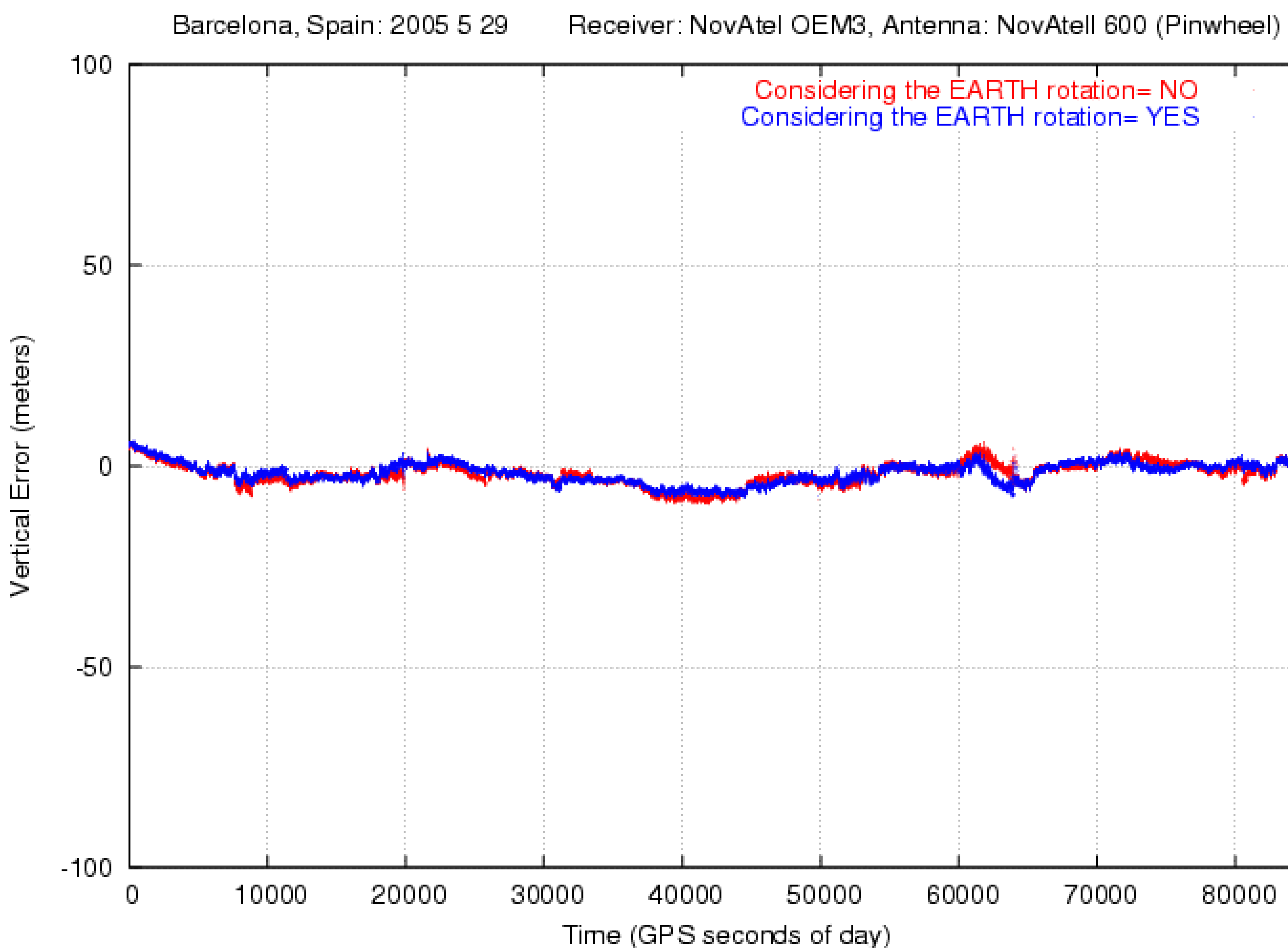
$$(X^{sat}, Y^{sat}, Z^{sat})_{CTS[reception]} = R_3(\omega_E\ \Delta t) . (X^{sat}, Y^{sat}, Z^{sat})_{CTS[emission]}$$

Variation in range: $\Delta\rho = \rho' - \rho_{emission}$

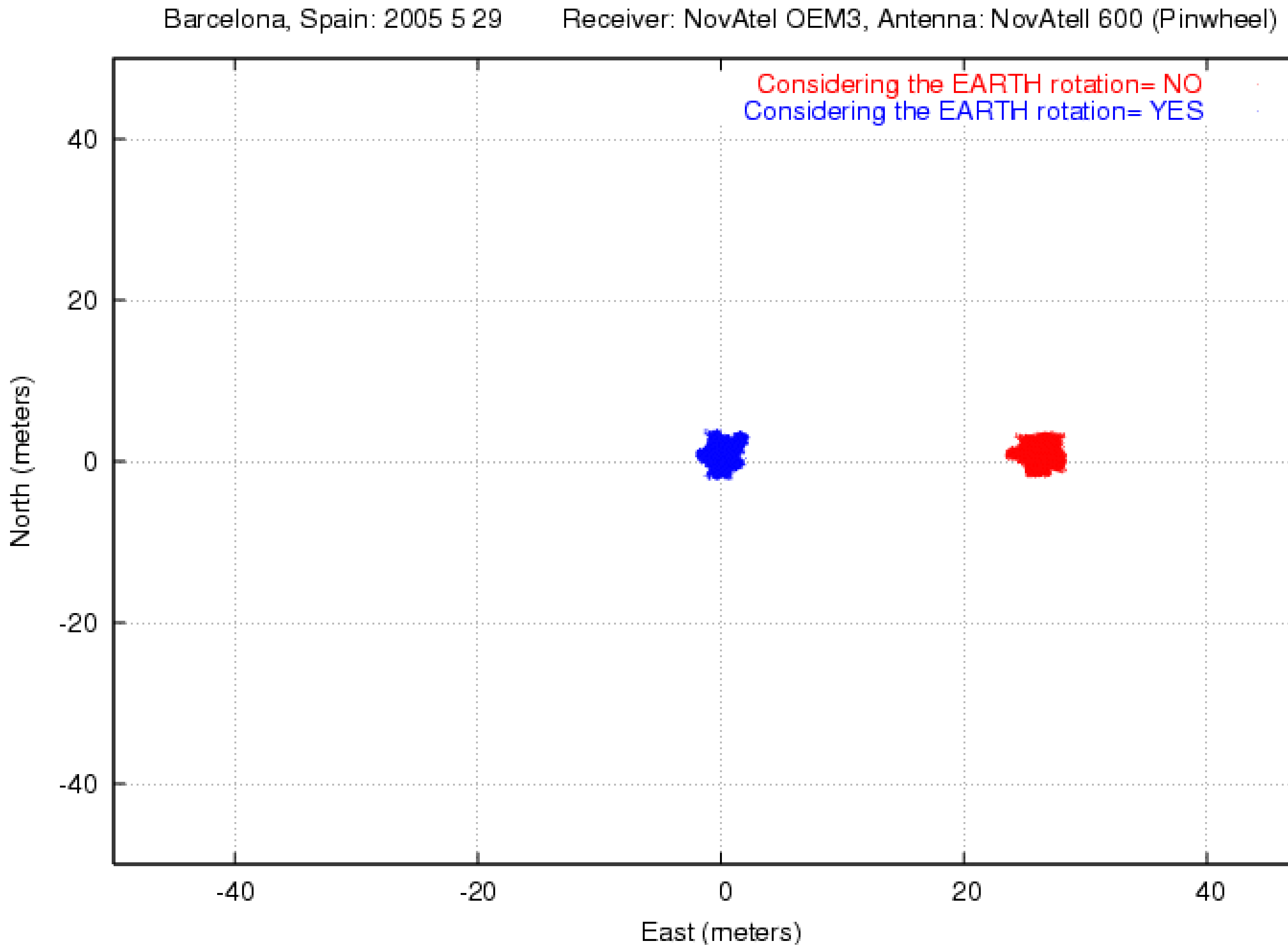




Vertical error comparison



Horizontal error comparison



Satellite and receiver clock offsets

- They are time-offsets between satellite/receiver time and GPS system time (provided by the ground control segment):

- The receiver clock offset (dt_{rec}) is estimated together with receiver coordinates.

- Satellite clock offset (dt^{sat}) may be computed from navigation message plus a Relativistic clock correction

$$dt^{sat} = a_0 + a_1(t - t_0) + a_2(t - t_0)^2 + \Delta rel^{sat}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(\overline{dt}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

dt^{sat}

$$a_0 + a_1(t-t_0) + a_2(t-t_0)^2$$

t0
YY MM DD H M S

a0 **a1** **a2**

PRN

```
 2              NAVIGATION DATA     GPS                 RINEX VERSION / TYPE
srx/v1.8.1.4        BAI                 95/10/19 03:18:35   PGM / RUN BY / DATE
CASA                                                        COMMENT
 -2444431.2031 -4428688.6270  3875750.1442                  COMMENT
                                                            END OF HEADER
14 95 10 18 00 51 44.0 1.129414886236D-05 1.136868377216D-13 0.000000000000D+00
    1.730000000000D+02-5.175000000000D+01 4.375182243902D-09-5.836427291652D-01
   -2.712011337280D-06 2.427505562082D-03 8.568167686462D-06 5.153718931198D+03
    2.623040000000D+05 4.470348358154D-08 1.698435481558D+00 1.676380634308D-08
    9.636381916043D-01 2.153437500000D+02 3.056960010495D+00-8.030691653399D-09
   -5.178787145843D-11 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00
    3.200000000000D+01 0.000000000000D+00 1.396983861923D-09 1.730000000000D+02
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```
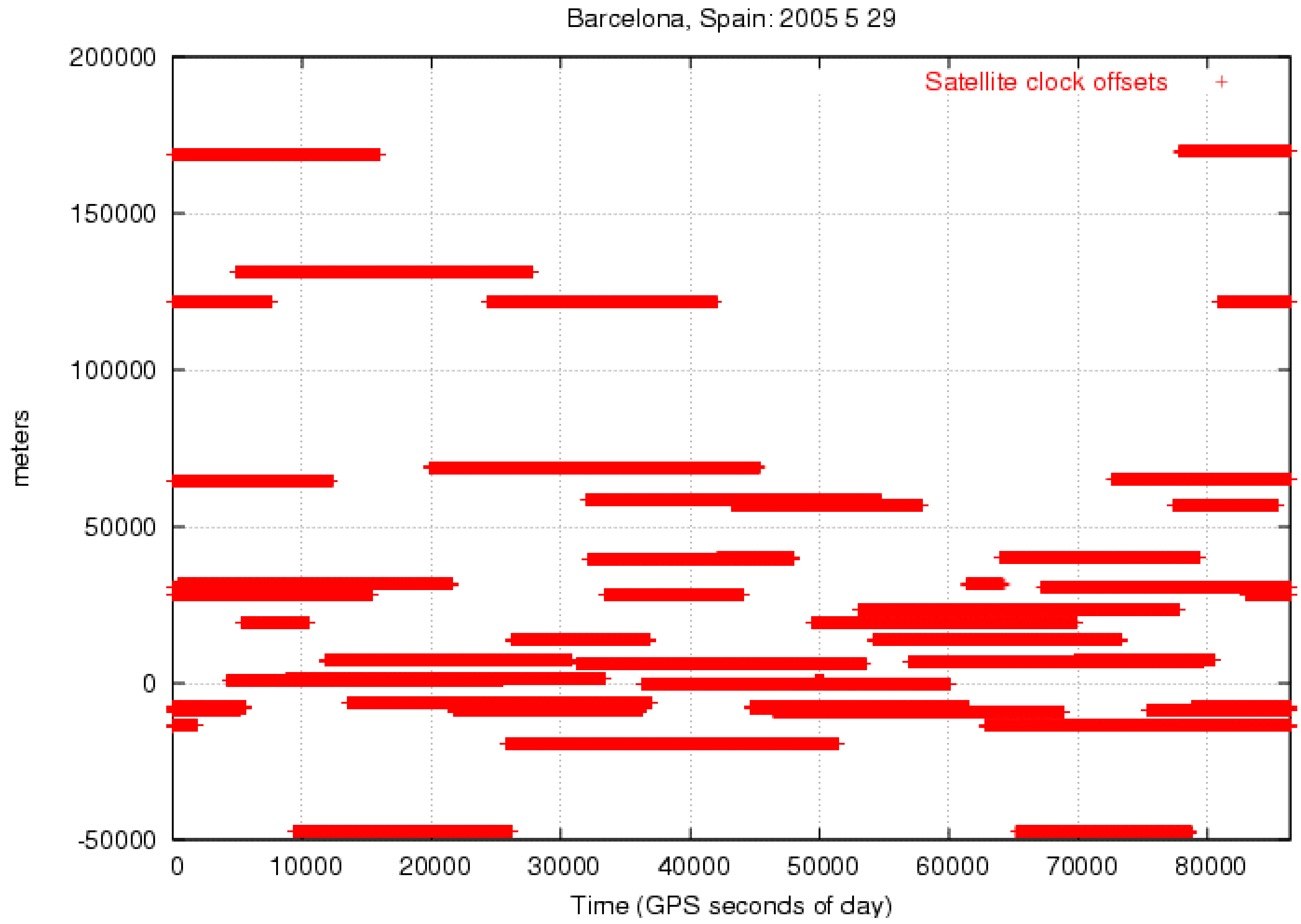
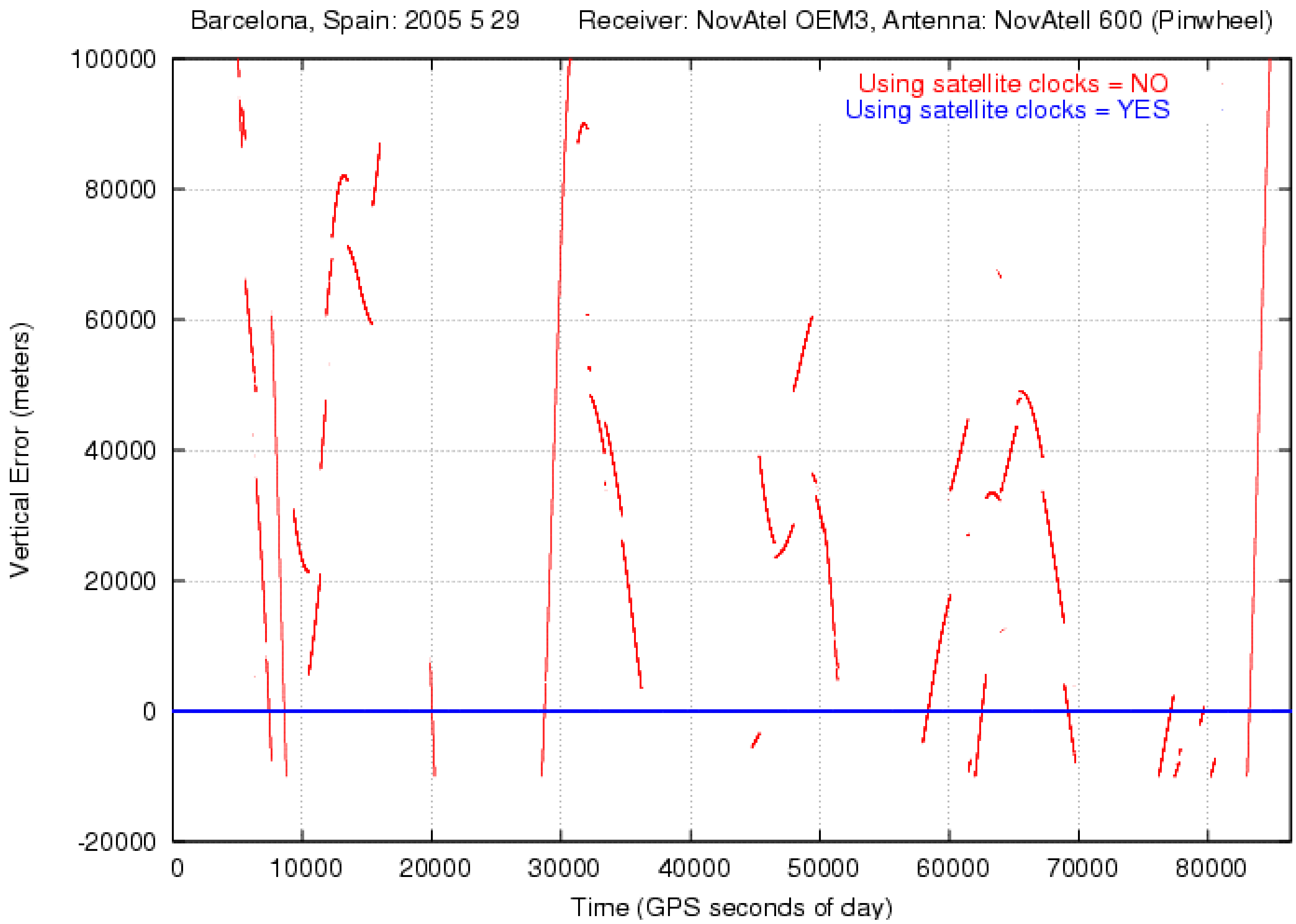

Range variation: satellite clocks



Vertical error comparison



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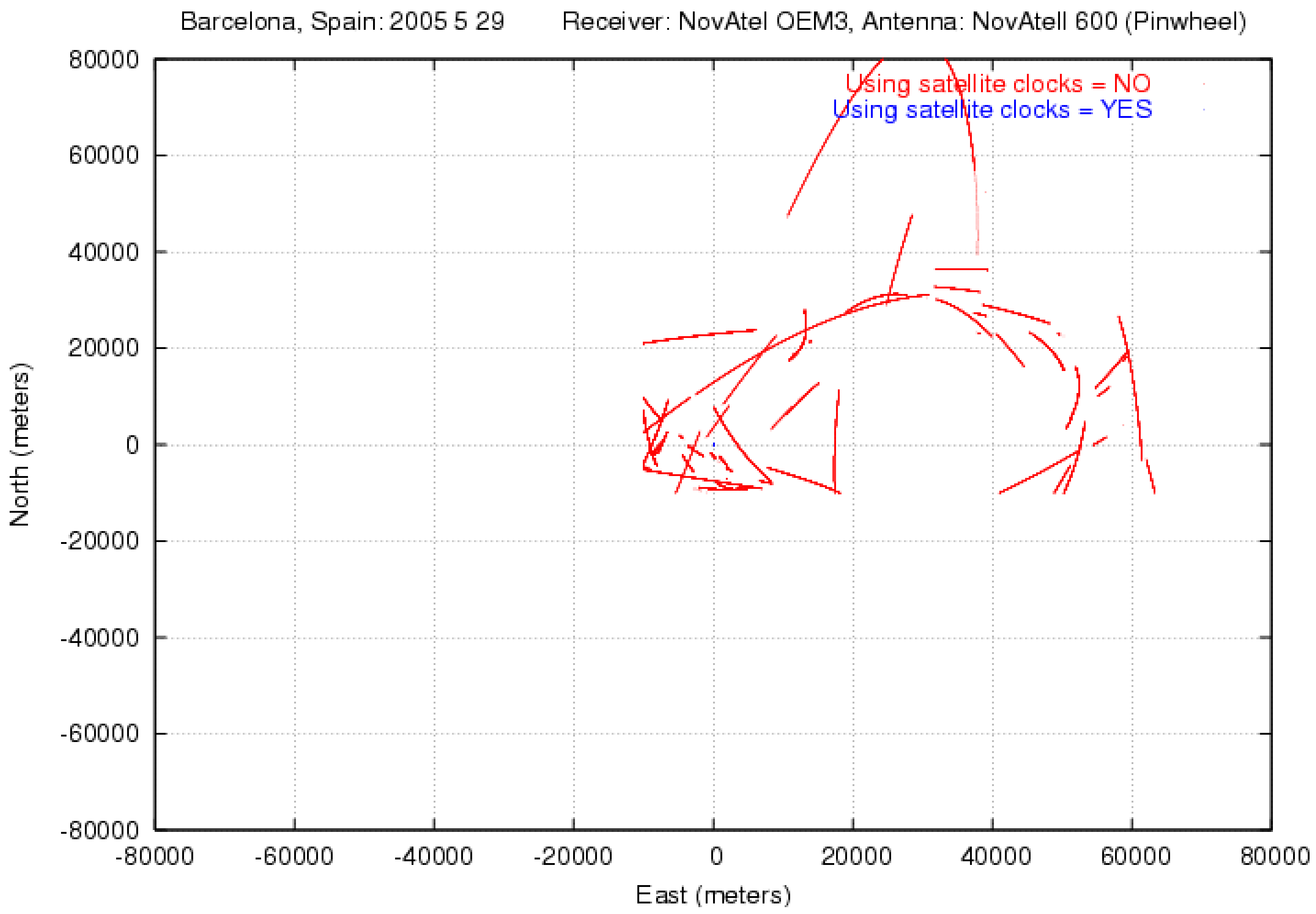
Horizontal error comparison



Relativistic clock correction (Δ_{rel})

- A constant component depending only on nominal value of satellite's orbit major semi-axis, being corrected modifying satellite's clock oscillator frequency*:

$$\frac{f_0^{'} - f_0}{f_0} = \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{\Delta U}{c^2} = -4.464 \cdot 10^{-10}$$

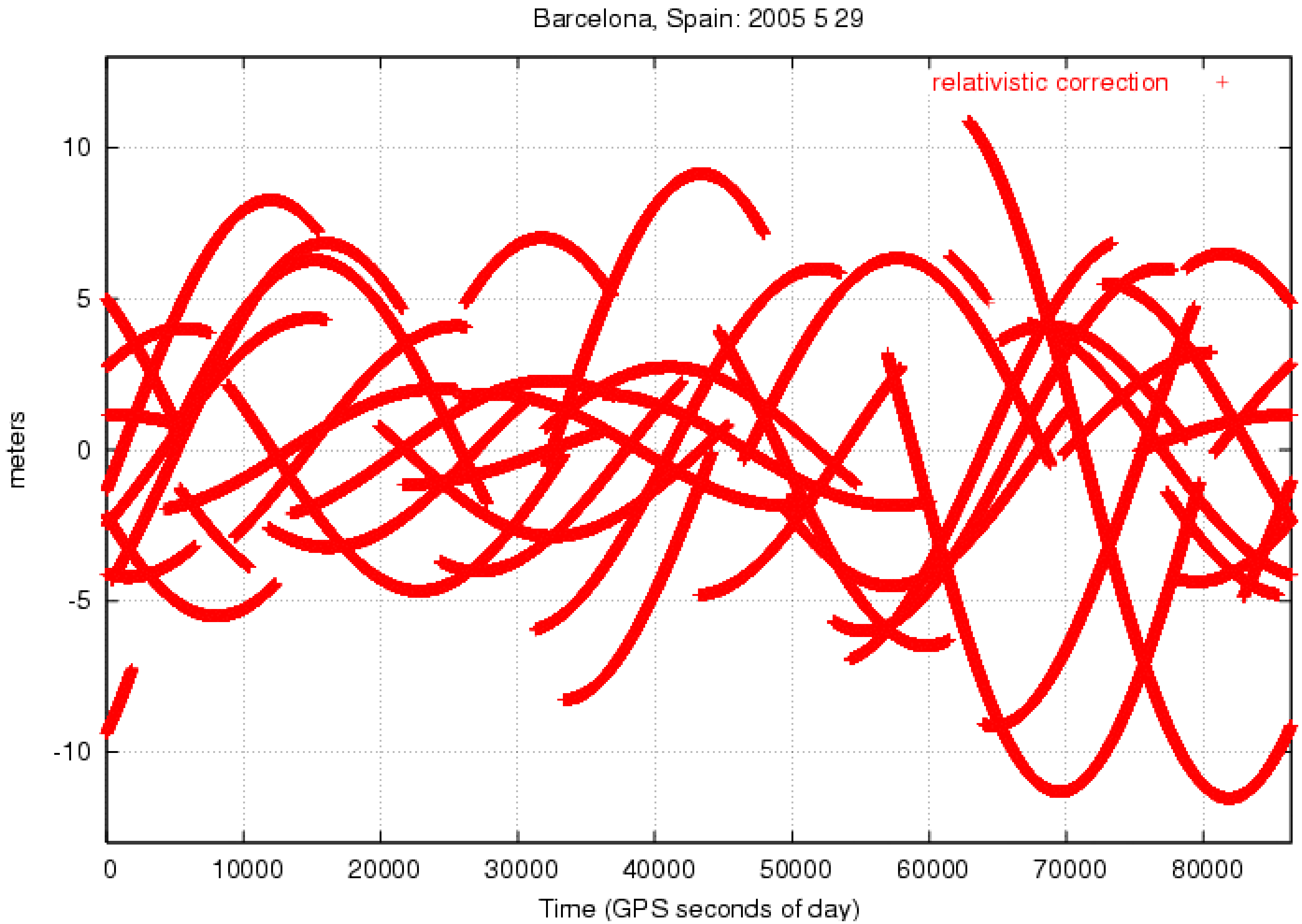
- A periodic component due to orbit eccentricity (to be corrected by user receiver):

$$\Delta_{rel} = -2\frac{\sqrt{\mu a}}{c^2} e \sin(E) = -2\frac{\mathbf{r} \cdot \mathbf{v}}{c^2} \; (seconds)$$

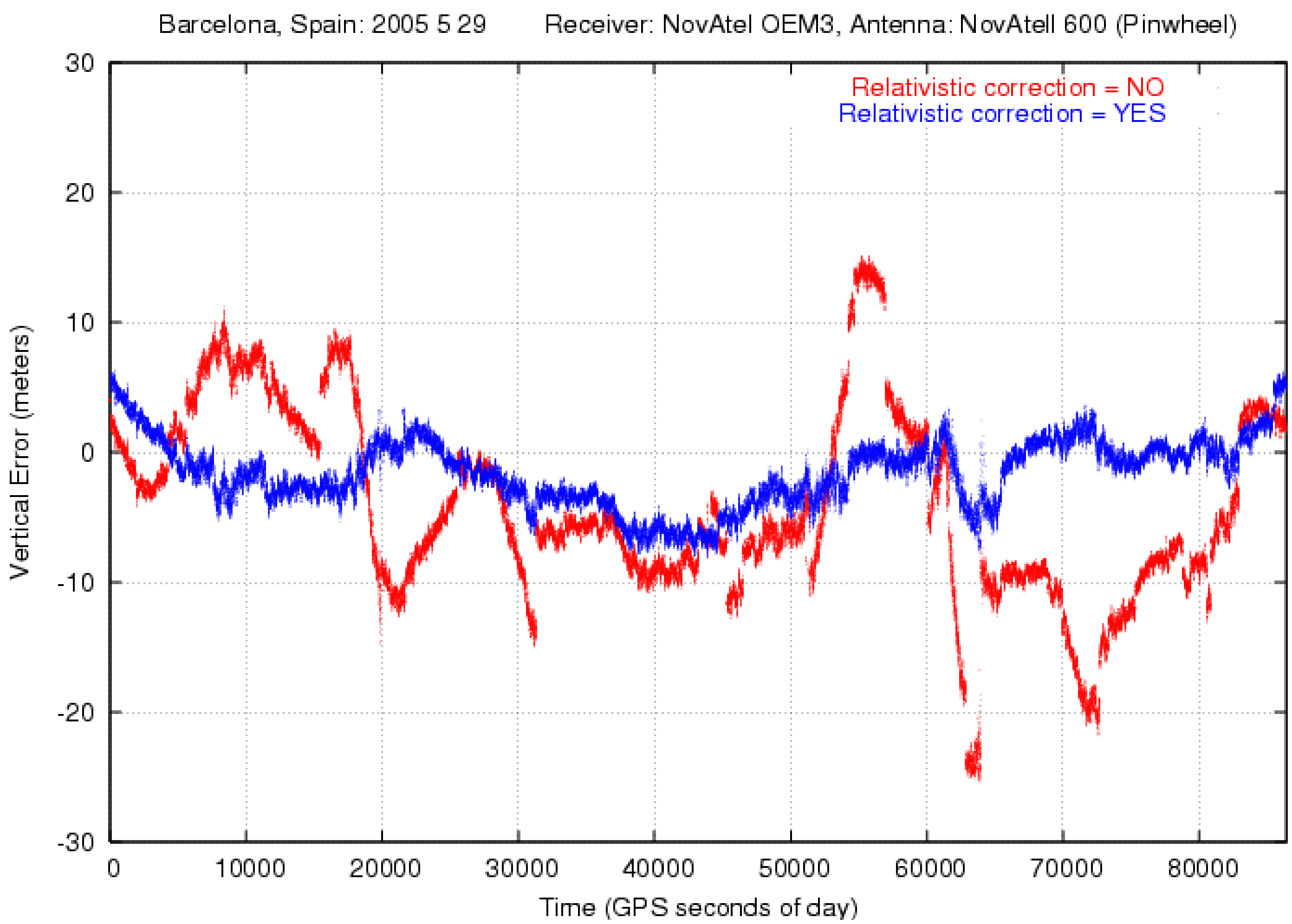
Being $\mu = 3.986005 \; 10^{14} \; (m^3/s^2)$ universal gravity constant, $c = 299792458 \; (m/s)$ light speed in vacuum, a is orbit's major semi-axis, e is its eccentricity, E is satellite's eccentric anomaly, and r and v are satellite's geocentric position and speed in an inertial system.

*being f_0 = 10.23 MHz, we have $\Delta f = 4.464 \; 10^{-10} \; f_0 = 4.57 \; 10^{-3}$ Hz so satellite should use $f'o = 10.22999999543$ MHz.

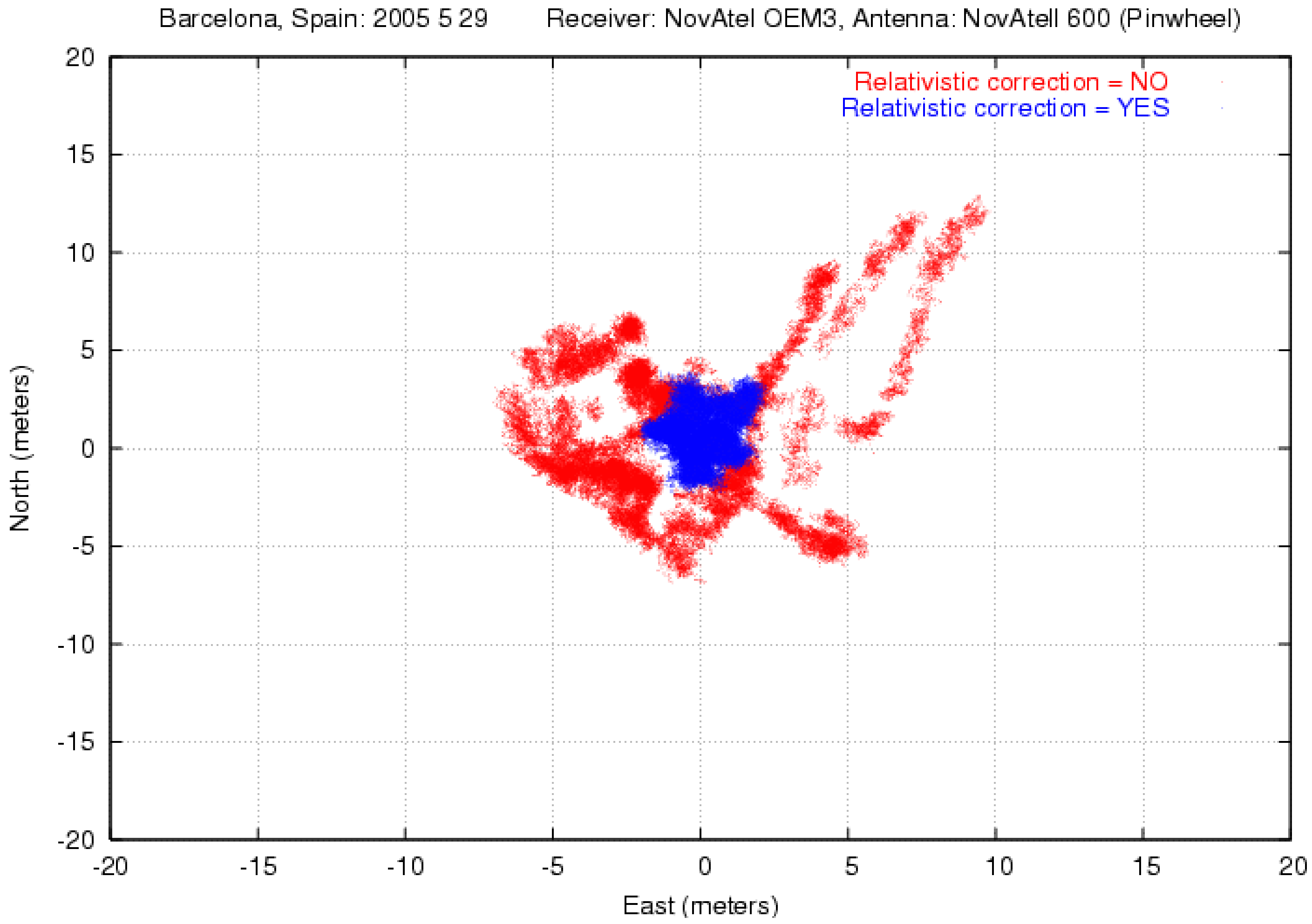
Range variation: relativistic correction



Vertical error comparison



Horizontal error comparison



Ionospheric Delay $Ion_f{}_{rec}^{sat}$

The ionosphere extends from about 60 km in height until more than 2000 km, with a sharp electron density maximum at around 350 km.

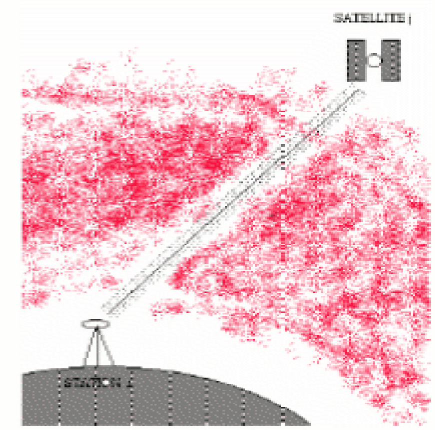
The ionosphere delays code and advances carrier by the same amount.

The ionospheric delay depends on signal frequency as given by:

$$Ion_1{}_{rec}^{sat} = \frac{40.3}{f_1^2} I$$

Where I is number of electrons per area unit in the direction of observation, or STEC (*Slant Total Electron Content*)

$$I = \int_{rec}^{sat} N_e \, ds$$



- For two-frequency receivers, it may be cancelled (99.9%) using ionosphere-free combination

$$LC = \frac{f_1^2 L1 - f_2^2 L2}{f_1^2 - f_2^2}$$

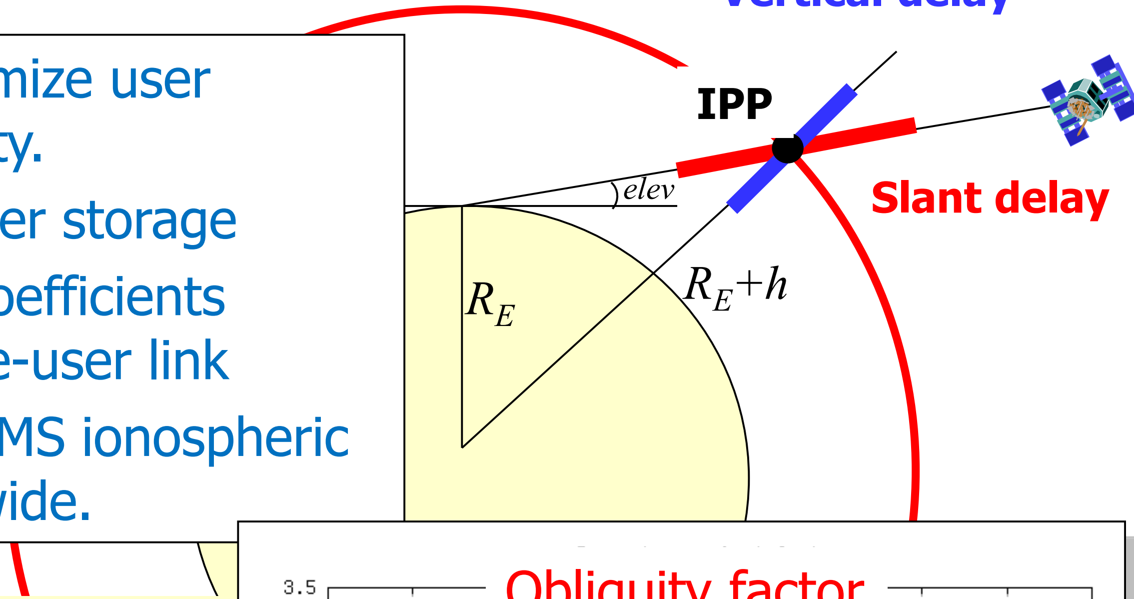
- For one-frequency receivers, it may be corrected (about 60%) using Klobuchar model (defined in GPS/SPS-SS), whose parameters are sent in navigation message. (See program klob.f)

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\bar{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

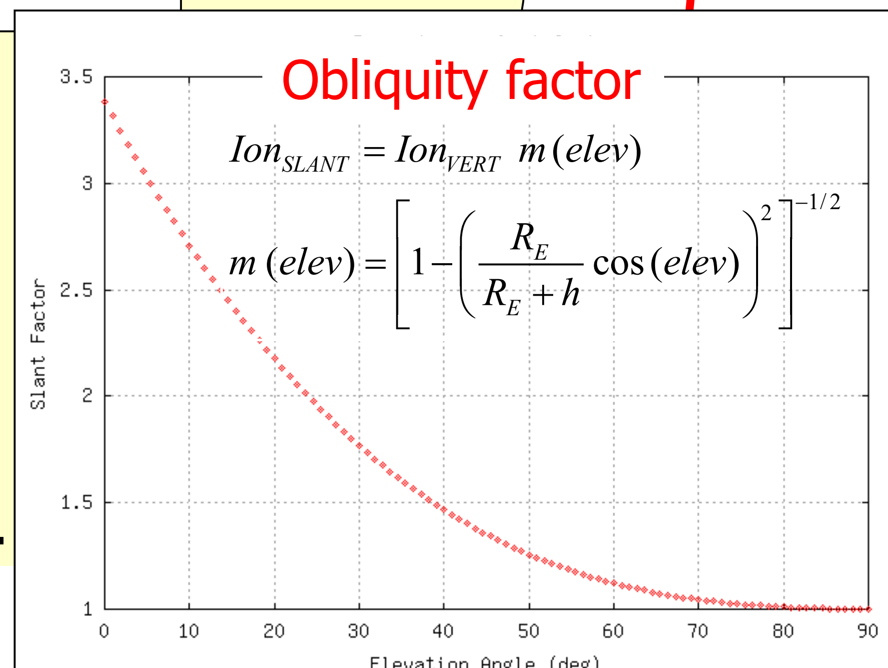
Klobuchar model (klob.f)

It was designed to minimize user computational complexity.

- Minimum user computer storage
- Minimum number of coefficients transmitted on satellite-user link
- At least 50% overall RMS ionospheric error reduction worldwide.



- It is assumed that the electron content is concentrated in a thin layer at 350km in height.
- The slant delay is computed from the vertical delay at the Ionospheric Pierce Point (IPP), multiplying by the obliquity factor.



$$Ion_{SLANT} = Ion_{VERT}\; m(elev)$$

$$m(elev) = \left[1 - \left(\frac{R_E}{R_E + h}\cos(elev)\right)^2\right]^{-1/2}$$

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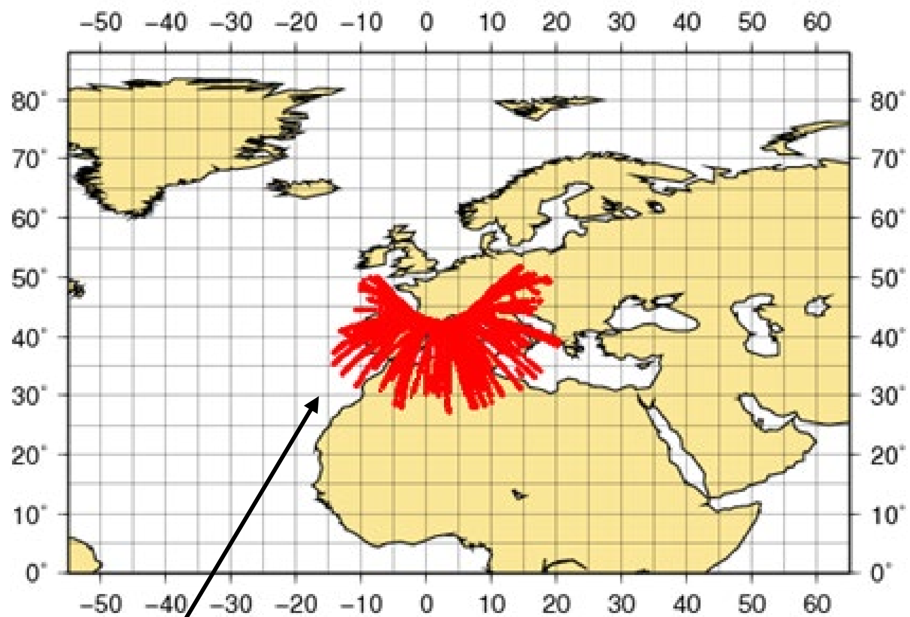
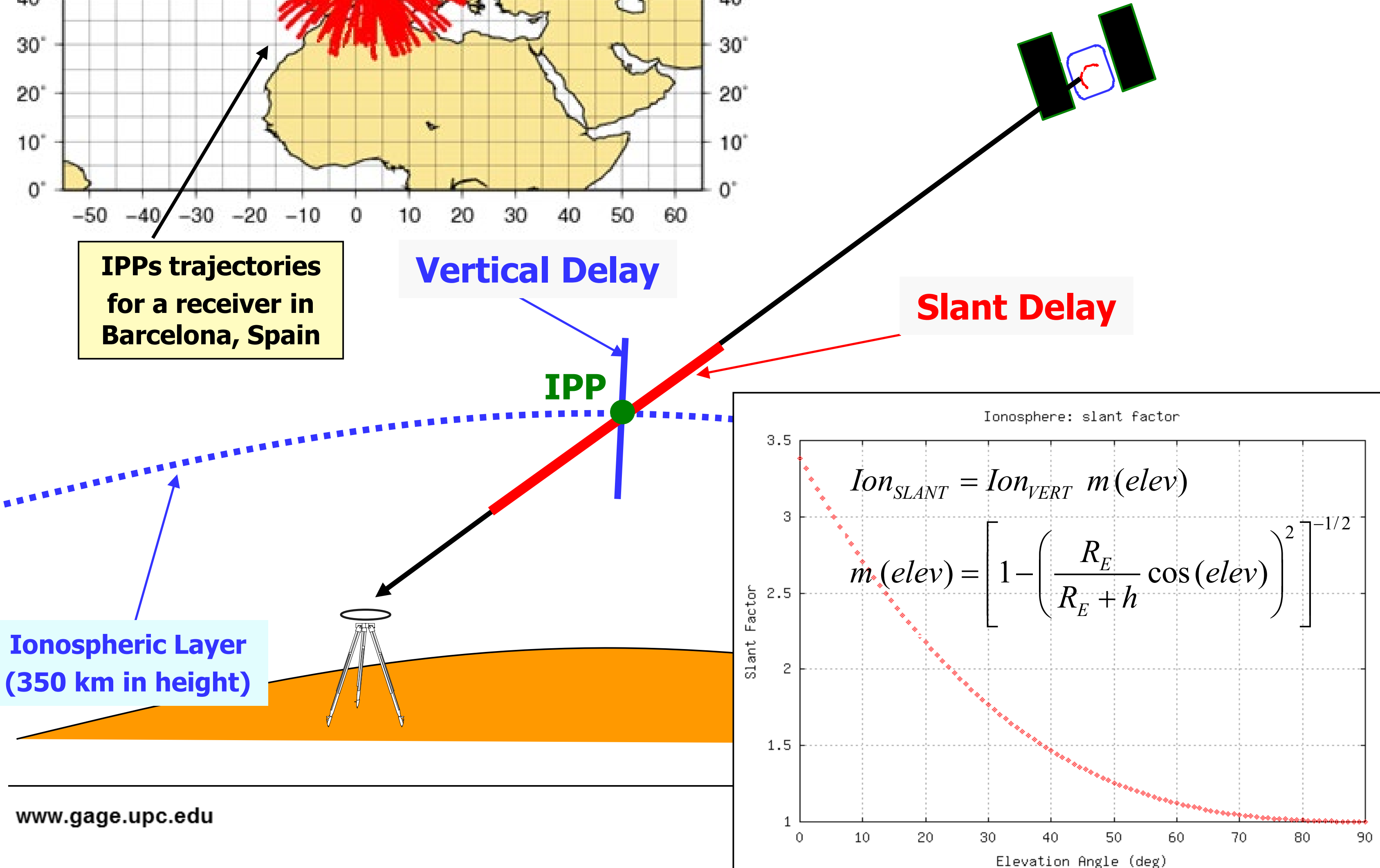
IONOSPHERIC PIERCE POINTS (IPP)





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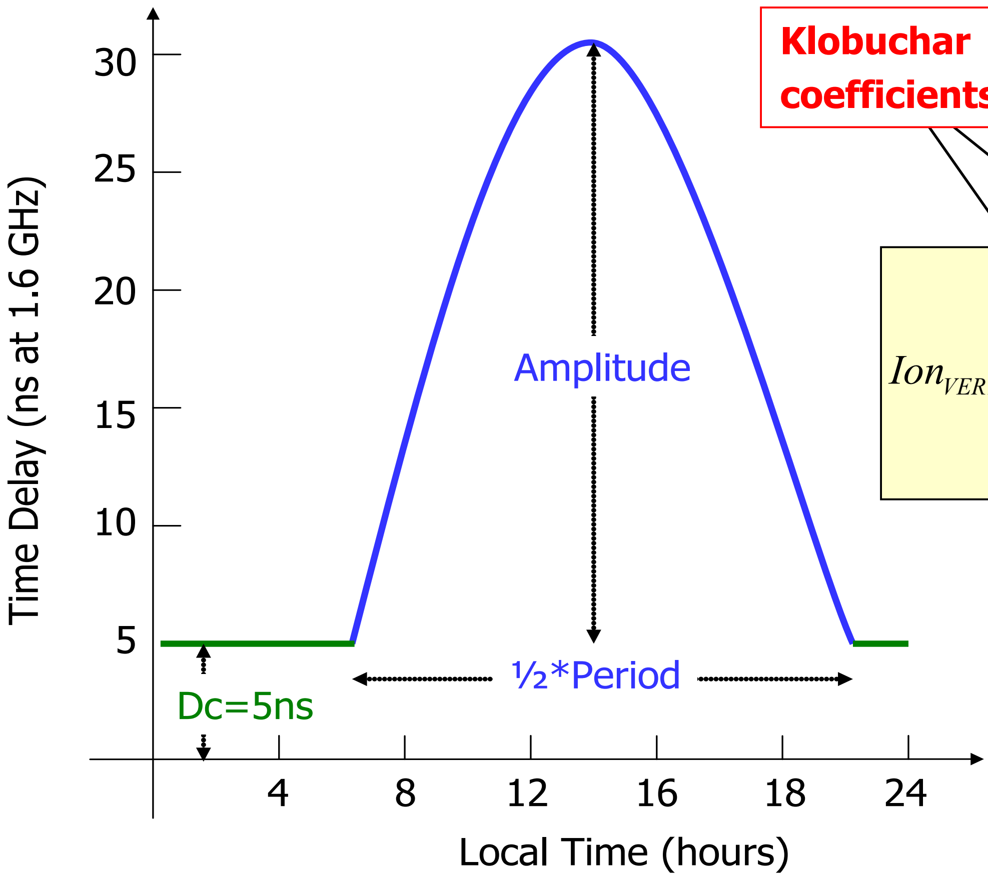
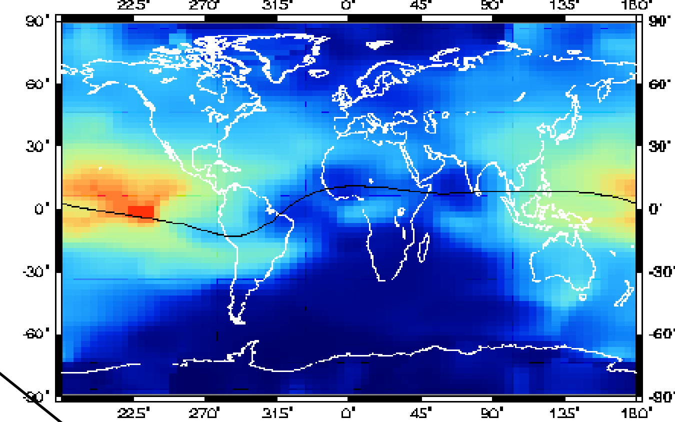
Klobuchar model



$$Ion_{SLANT} = Ion_{VERT}\ m(elev)$$

$$m(elev) = \left[1 - \left(\frac{R_E}{R_E + h}\cos(elev)\right)^2\right]^{-1/2}$$



Klobuchar coefficients

$$Ion_{VERT} = \begin{cases} DC + A\cos\left[\frac{2\pi(t-\Phi)}{P}\right] & (day) \\ DC \quad ;\ if \left[\frac{2\pi(t-\Phi)}{P}\right] > \frac{\pi}{2} & (night) \end{cases}$$

Being:

$$A = \sum_{n=0}^{3} \alpha_n \varphi^n \quad ; \quad P = \sum_{n=0}^{3} \beta_n \varphi^n$$

φ = Geomagnetic Latitude

Where:

Dc= 5ns

Φ= 14 **(ctt. phase offset)**

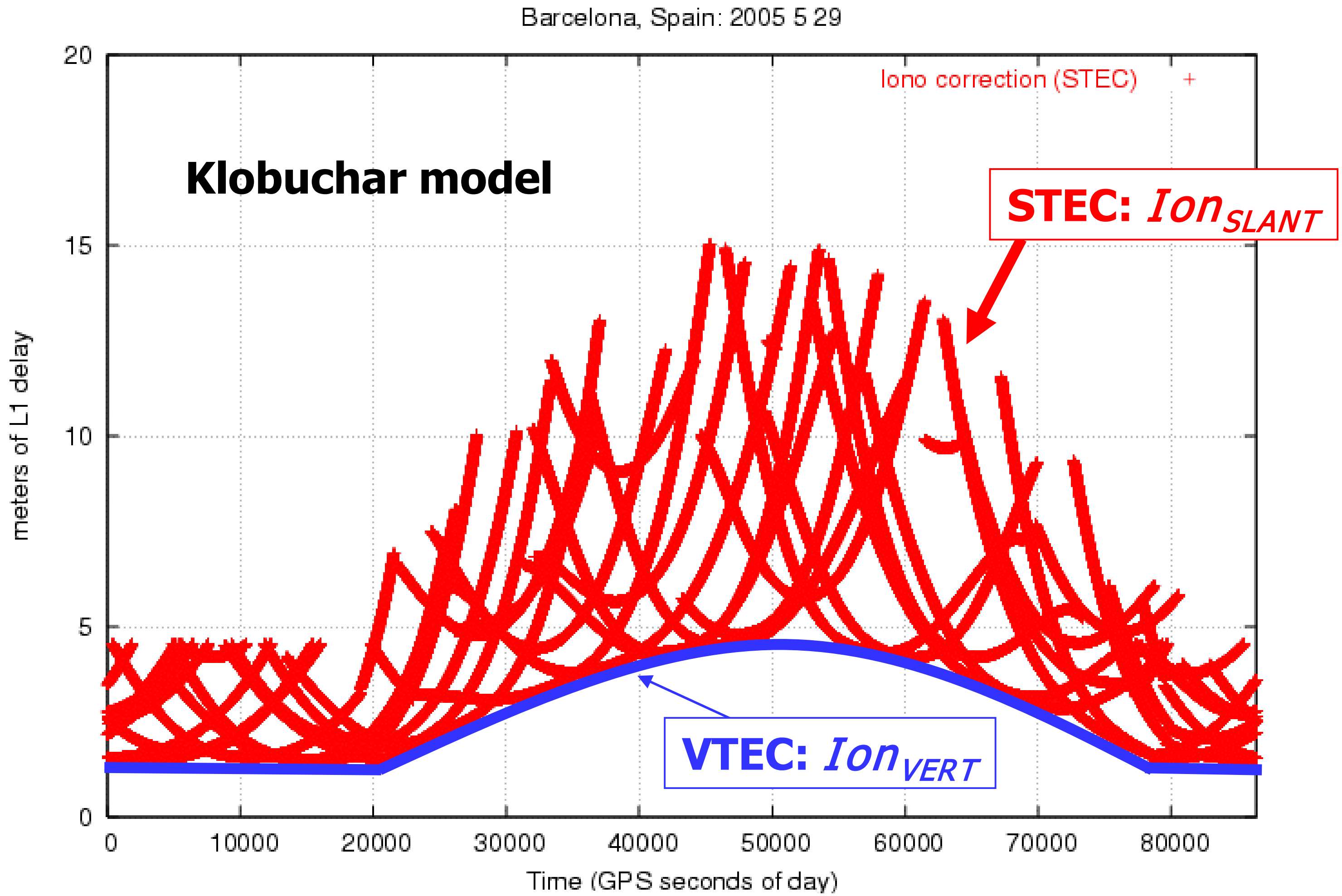
t = Local Time

$(time, r_{sta}, r^{sat}, \alpha 0, \alpha 1, \alpha 2, \alpha 3, \beta 0, \beta 1, \beta 2, \beta 3) \rightarrow [Klob] \rightarrow Iono$

elev, ϕ

```
 2              NAVIGATION DATA                        RINEX VERSION / TYPE
CCRINEXN V1.5.2 UX  CDDIS               24-MAR- 0 00:23     PGM / RUN BY / DATE
IGS BROADCAST EPHEMERIS FILE                                COMMENT
    0.3167D-07  0.4051D-07 -0.2347D-06  0.1732D-06          ION ALPHA
   -0.2842D+05 -0.2150D+05 -0.1096D+06  0.4301D+06          ION BETA
   -0.121071934700D-07-0.488498130835D-13   319488     1002 DELTA-UTC: A0,A1,T,W
    13                                                      LEAP SECONDS
                                                            END OF HEADER
 1 99  3 23  0  0  0.0 0.783577561379D-04 0.113686837722D-11 0.000000000000D+00
    0.191000000000D+03-0.106250000000D+01 0.487163149444D-08-0.123716752769D+01
   -0.540167093277D-07 0.476544268895D-02 0.713579356670D-05 0.515433833885D+04
    0.172800000000D+06-0.260770320892D-07-0.850753478531D+00 0.763684511185D-07
    0.957259887797D+00 0.241437500000D+03-0.167990552187D+01-0.823998608564D-08
    0.174650132022D-09 0.100000000000D+01 0.100200000000D+04 0.000000000000D+00
    0.320000000000D+02 0.000000000000D+00 0.465661287308D-09 0.191000000000D+03
    0.172800000000D+06 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
```

Range variation: Ionospheric correction



$$Ion_{SLANT} = Ion_{VERT} \; m(elev)$$

$$m(elev) = \left[1 - \left(\frac{R_E}{R_E + h} \cos(elev) \right)^2 \right]^{-1/2}$$

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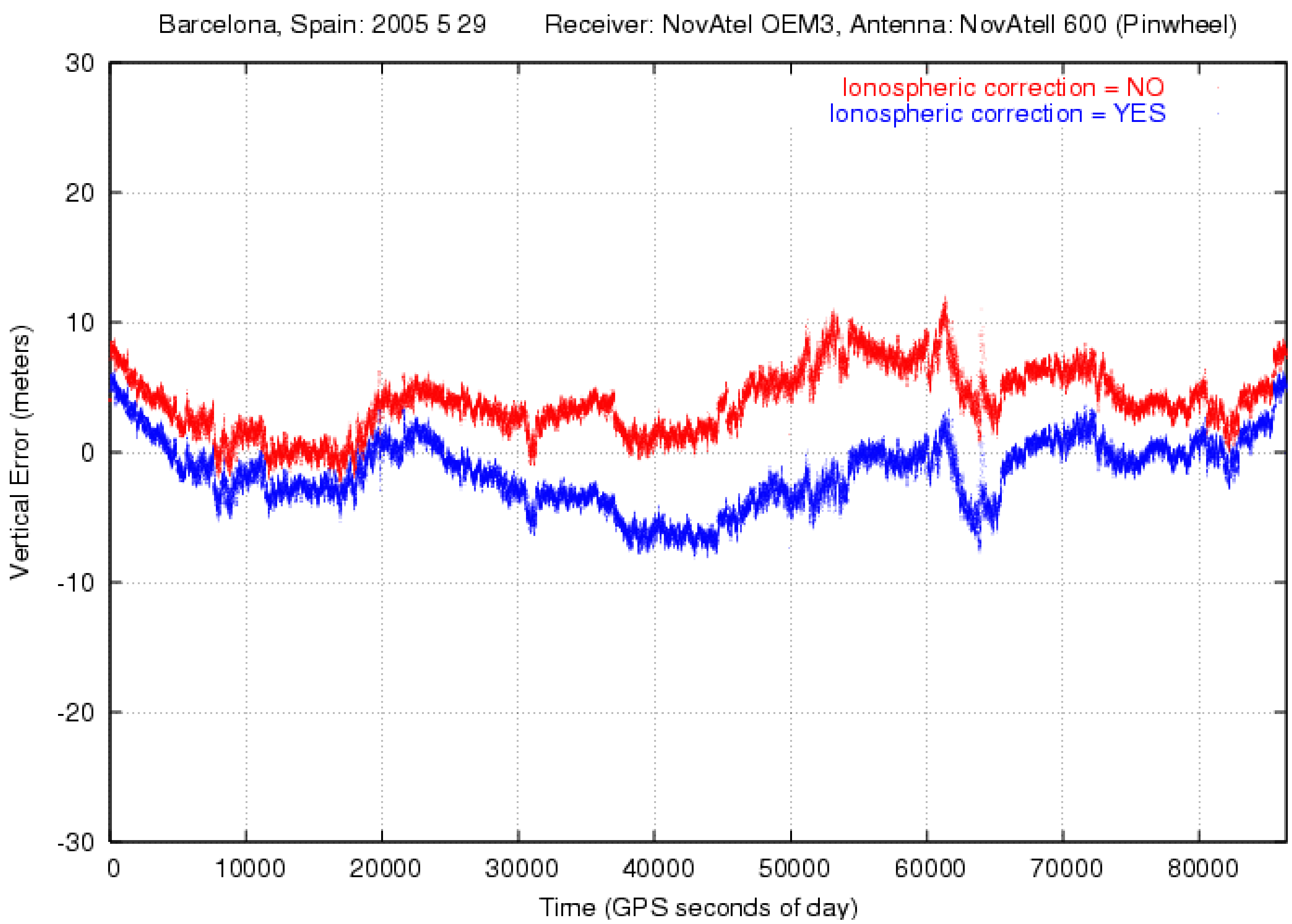
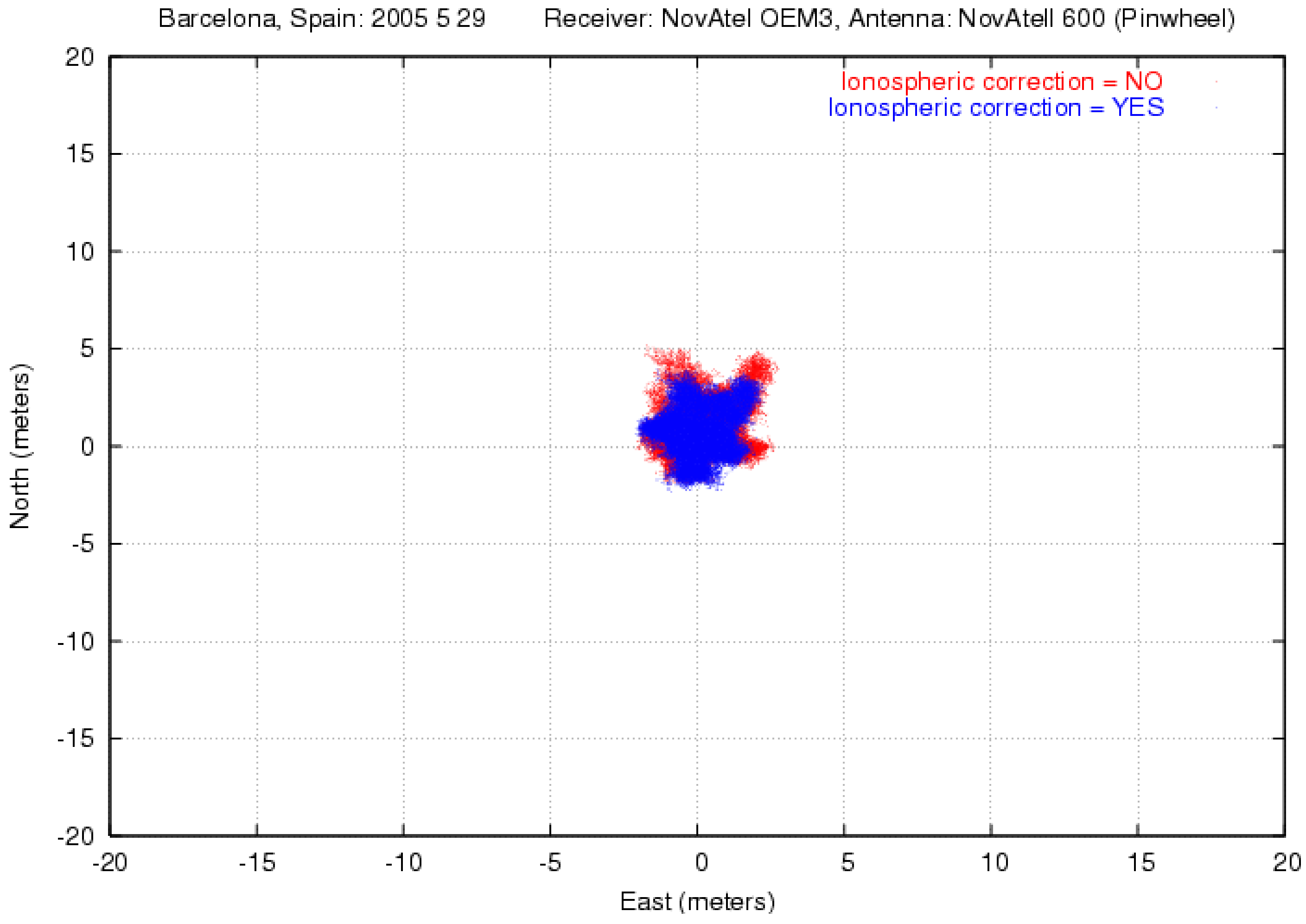
Vertical error comparison



Horizontal error comparison



Galileo Single Frequ. Ionospheric Corr. Algo. (NeQuick model)

SENSOR STATION

Observe slant TEC in Galileo Sensor Stations for 24 hours

Optimise effective ionisation parameter for NeQuick to match observations

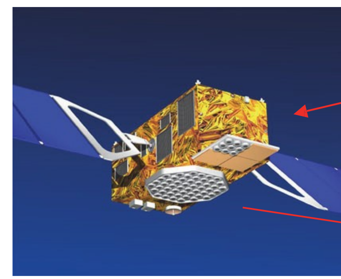
SATELLITE

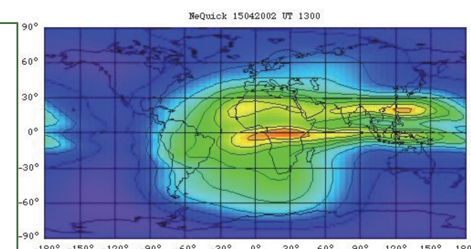
Transmit effective ionisation parameter in Galileo Navigation message

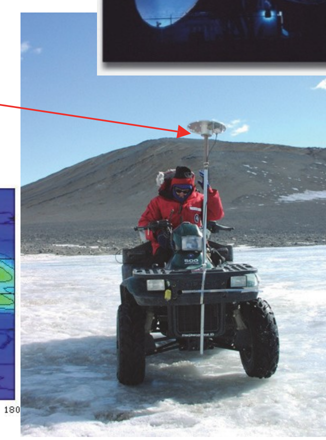
$$Az = a_0 + a_1 \cdot \mu + a_2 \cdot \mu^2$$

USER RECEIVER

Calculate slant TEC using NeQuick with broadcast ionisation parameter.
Correct for Ionospheric delay at frequency in question.



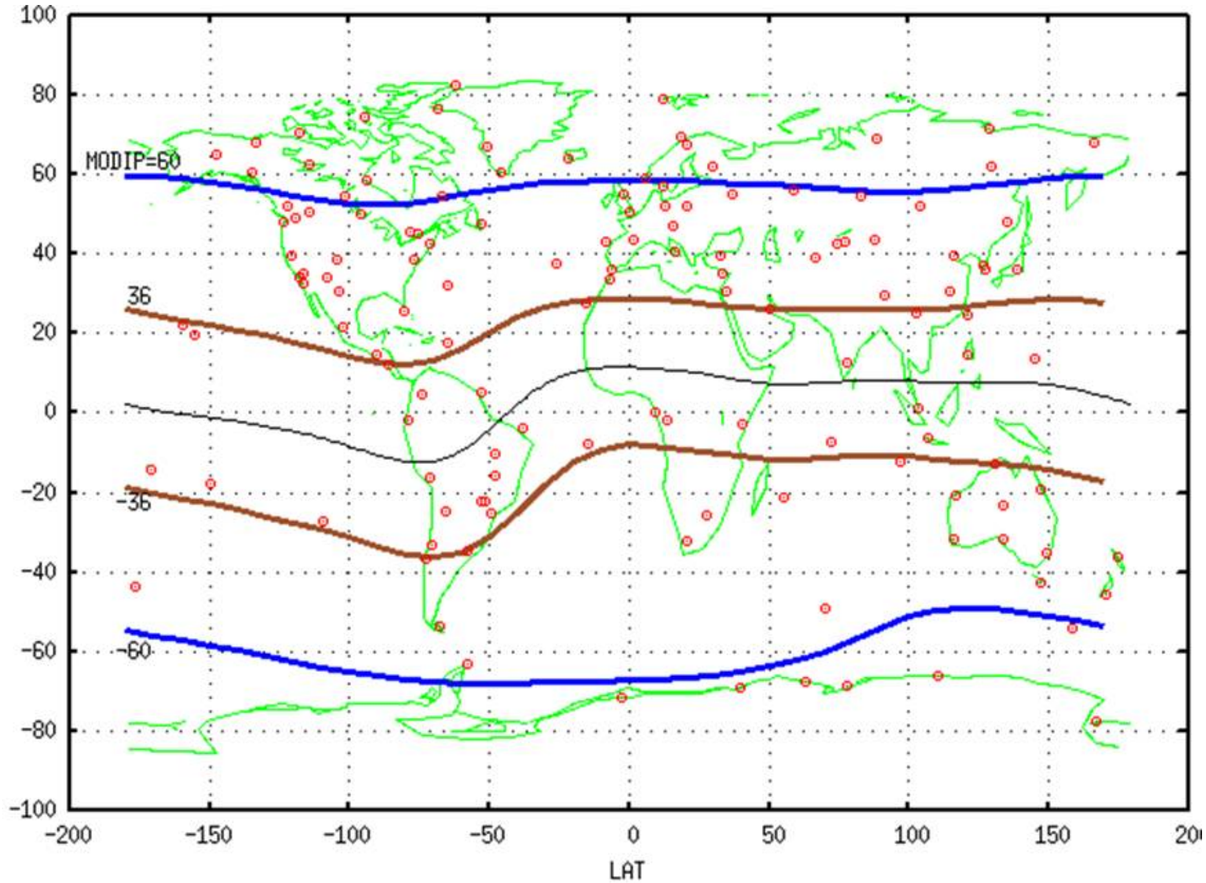




μ is the Modified DIP latitude (**MODIP**)

Source: Bertram Arbesser-Rastburg
ESA/JRC International Summer School on GNSS 2016

MODIP bounds



MOdied DIP latitude (MODIP) μ,

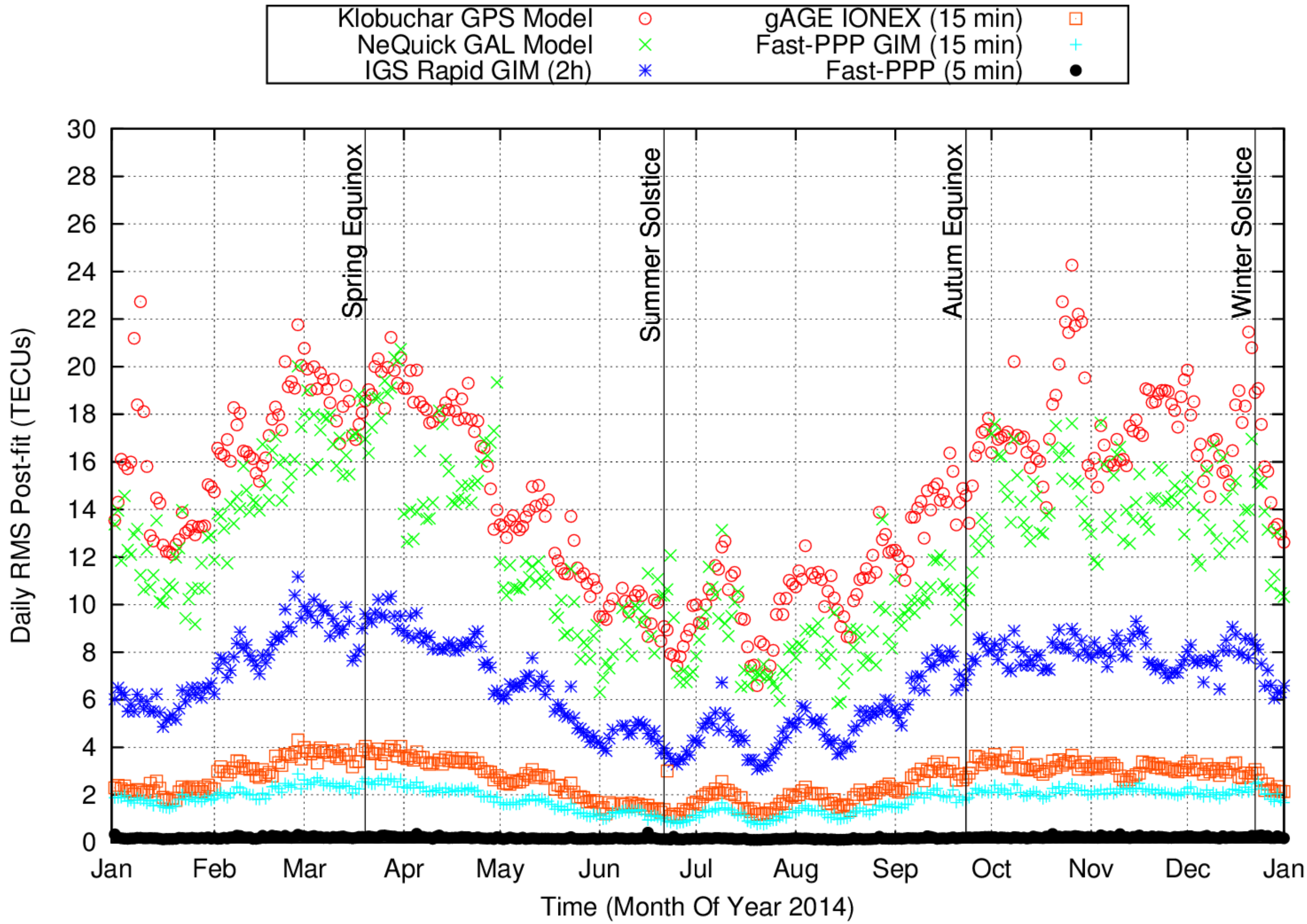
$$\tan \mu = \frac{I}{\sqrt{\cos \varphi}}$$

with I the true magnetic inclination, or ***dip*** in the ionosphere (usually at 300 km), and φ the geographic latitude of the receiver.

Ionospheric models used by the GNSSs

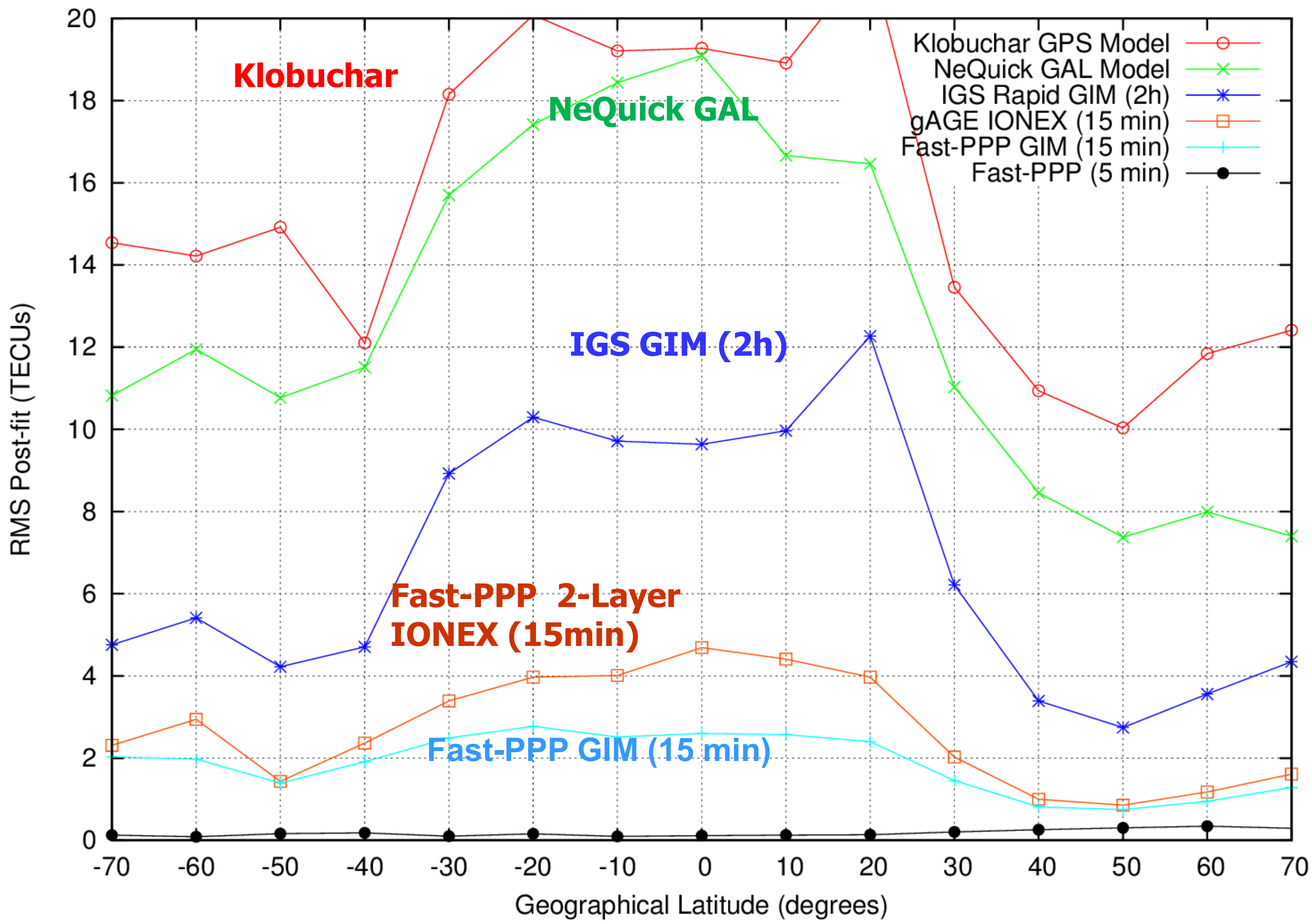
GPS	Klobuchar model
GLONASS	No ionospheric model is broadcasted
BeiDou	Klobuchar model (with layer height at 375km instead of 350km)
Galileo	NeQuick model

Ionospheric models performance comparison



Picture from [RD-5]

Ionospheric models performance comparison



Picture from [RD-5]

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Tropospheric Delay

Troposphere is the atmospheric layer placed between Earth's surface and an altitude of about 60km.

The tropospheric delay does not depend on frequency and affects both the code and carrier phases in the same way. It can be modeled (about 90%) as:

- d_{dry} corresponds to the vertical delay of the dry atmosphere (basically oxygen and nitrogen in hydrostatical equilibrium)
  ➔ It can be modeled as an ideal gas.
- d_{wet} corresponds to the vertical delay of the wet component (water vapor) ➔ difficult to model.

A simple model is:

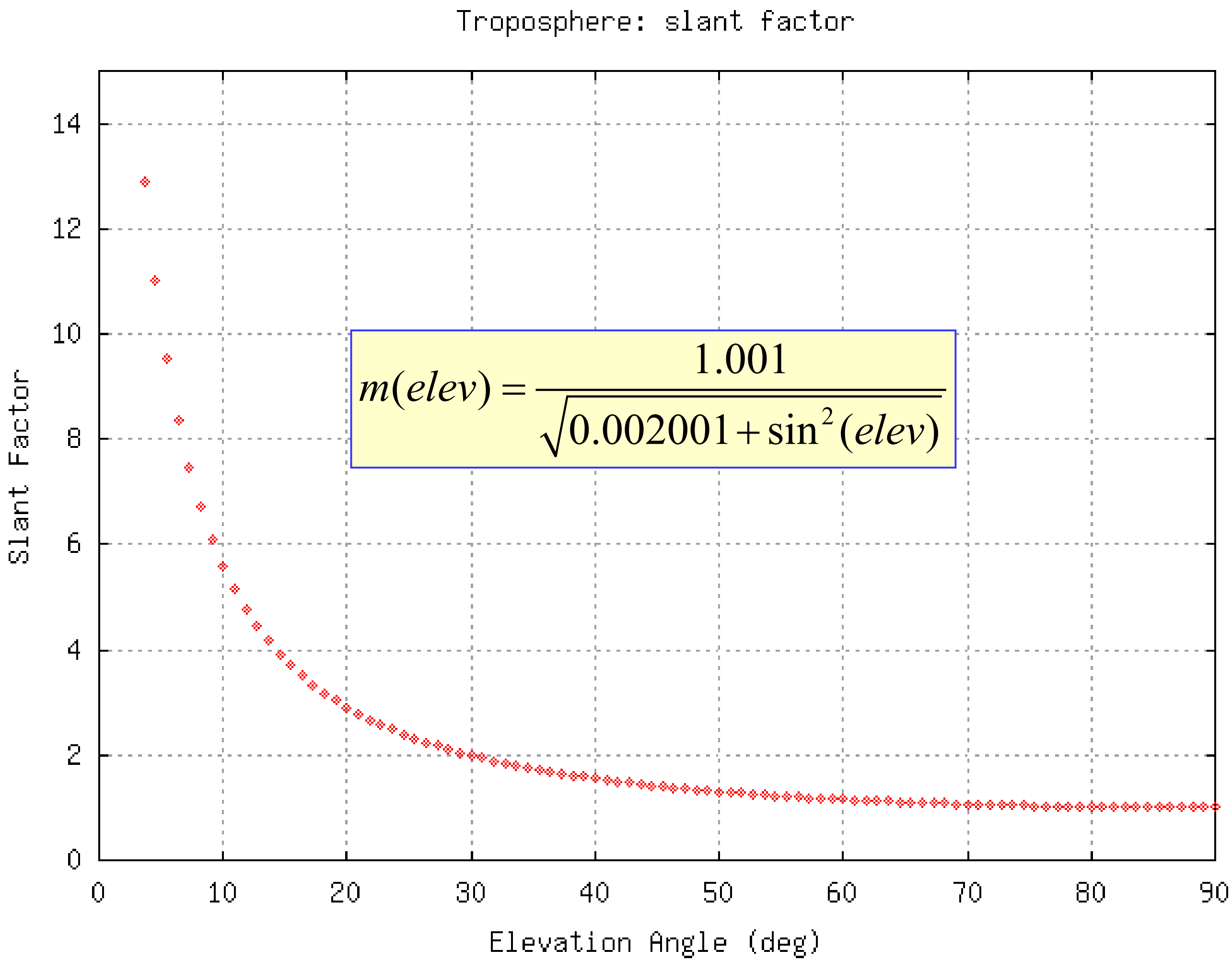
$$Trop_{rec}^{sat} = (d_{dry} + d_{wet}) \cdot m(elev)$$

$$d_{dry} = 2.3 \exp(-0.116 \cdot 10^{-3} H) \;\; meters$$

$$d_{wet} = 0.1 m \quad [H : height\ over\ the\ sea\ level]$$

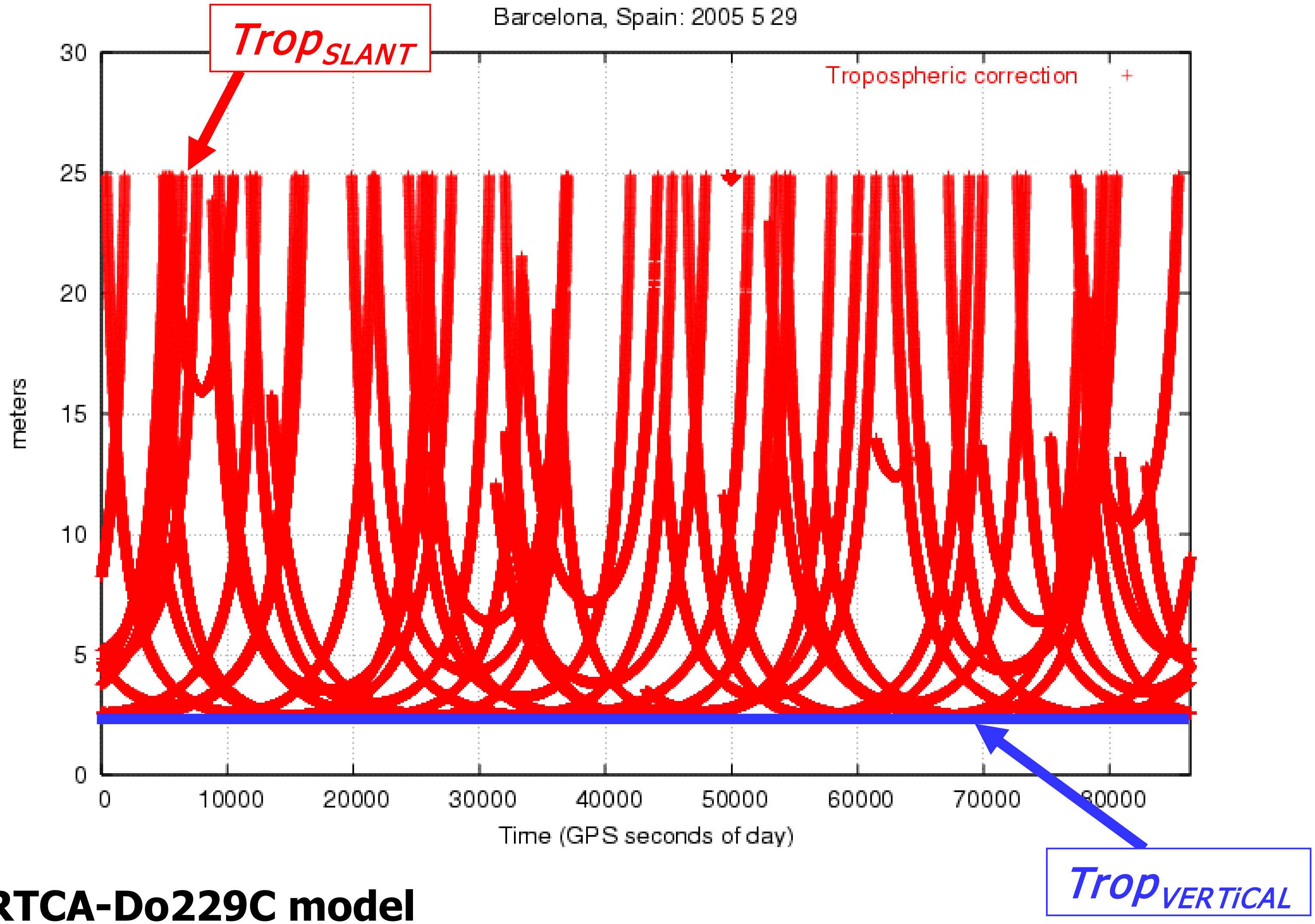
$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\bar{t}^{\,sat} + \Delta rel^{sat}\right) + \boxed{Trop_{rec}^{sat}} + Ion_{1rec}^{sat} + TGD^{sat}$$



$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

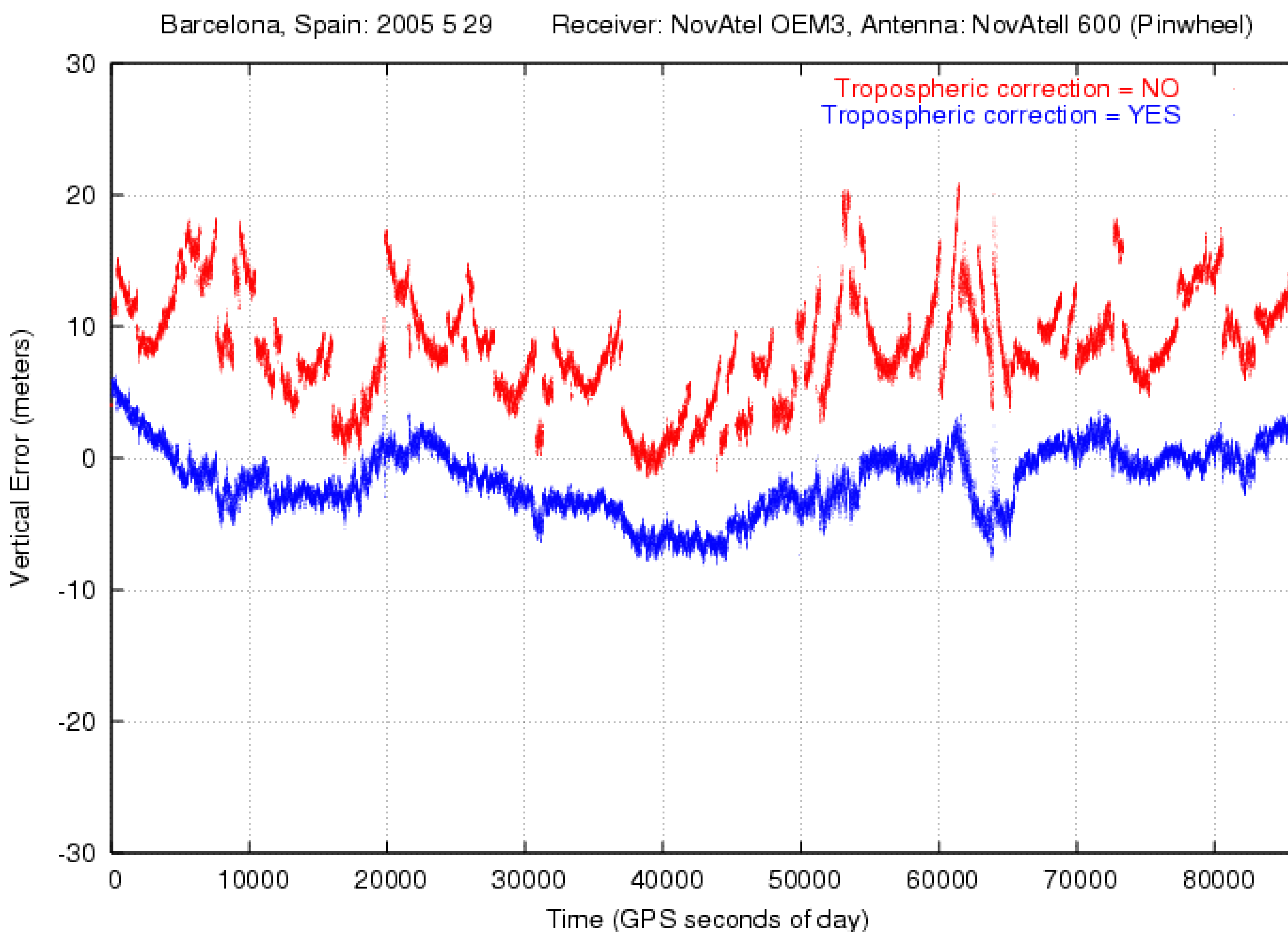
Range variation: Tropospheric correction



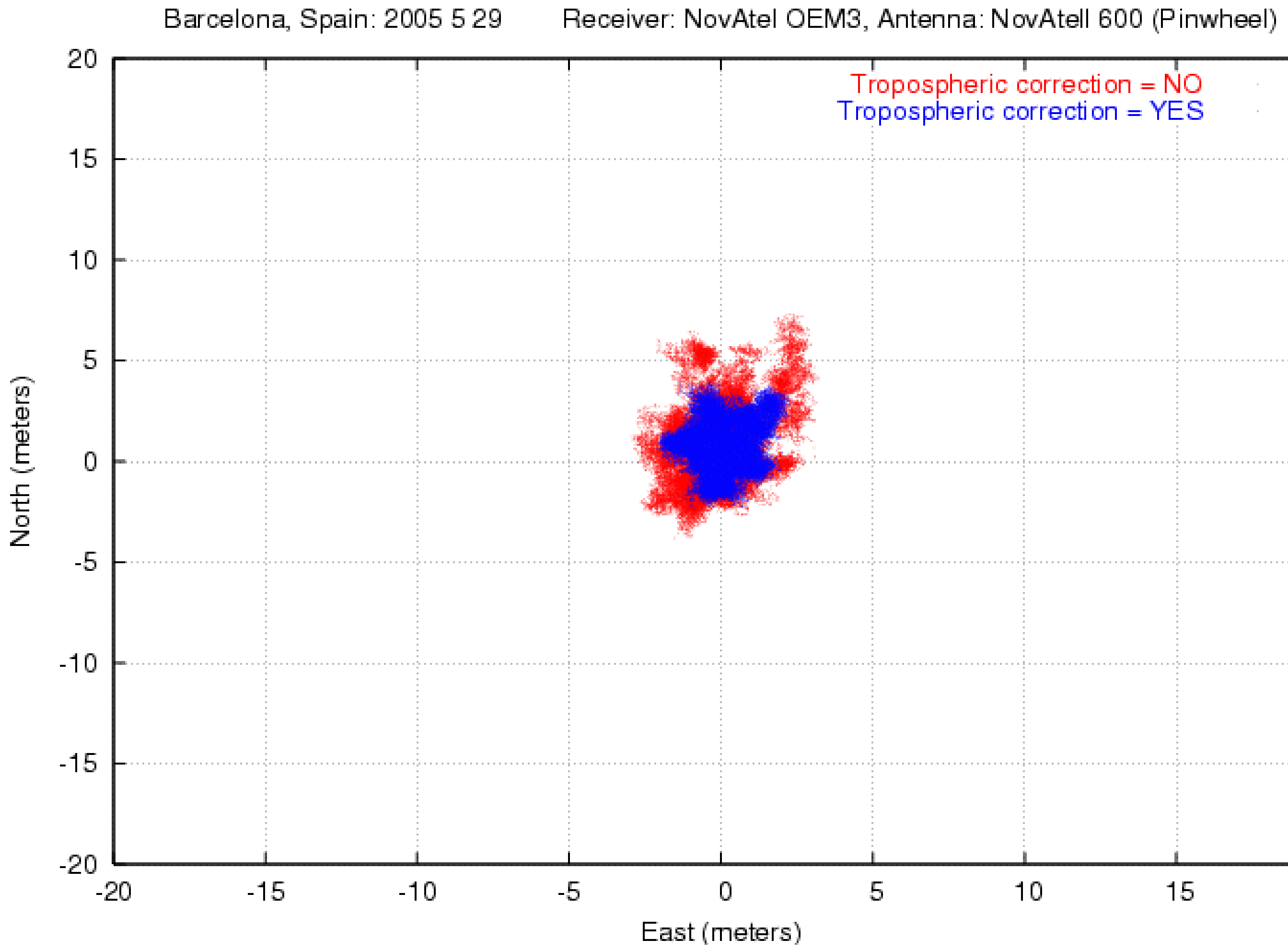
RTCA-Do229C model

gAGE

Vertical error comparison



Horizontal error comparison



Instrumental Delays

Some sources for these delays are antennas, cables, as well as several filters used in both satellites and receivers.

They are composed by a delay corresponding to satellite and other to receiver, depending on frequency:

$$K_{1,rec}^{sat} = K_{1,rec} + TGD^{sat}$$

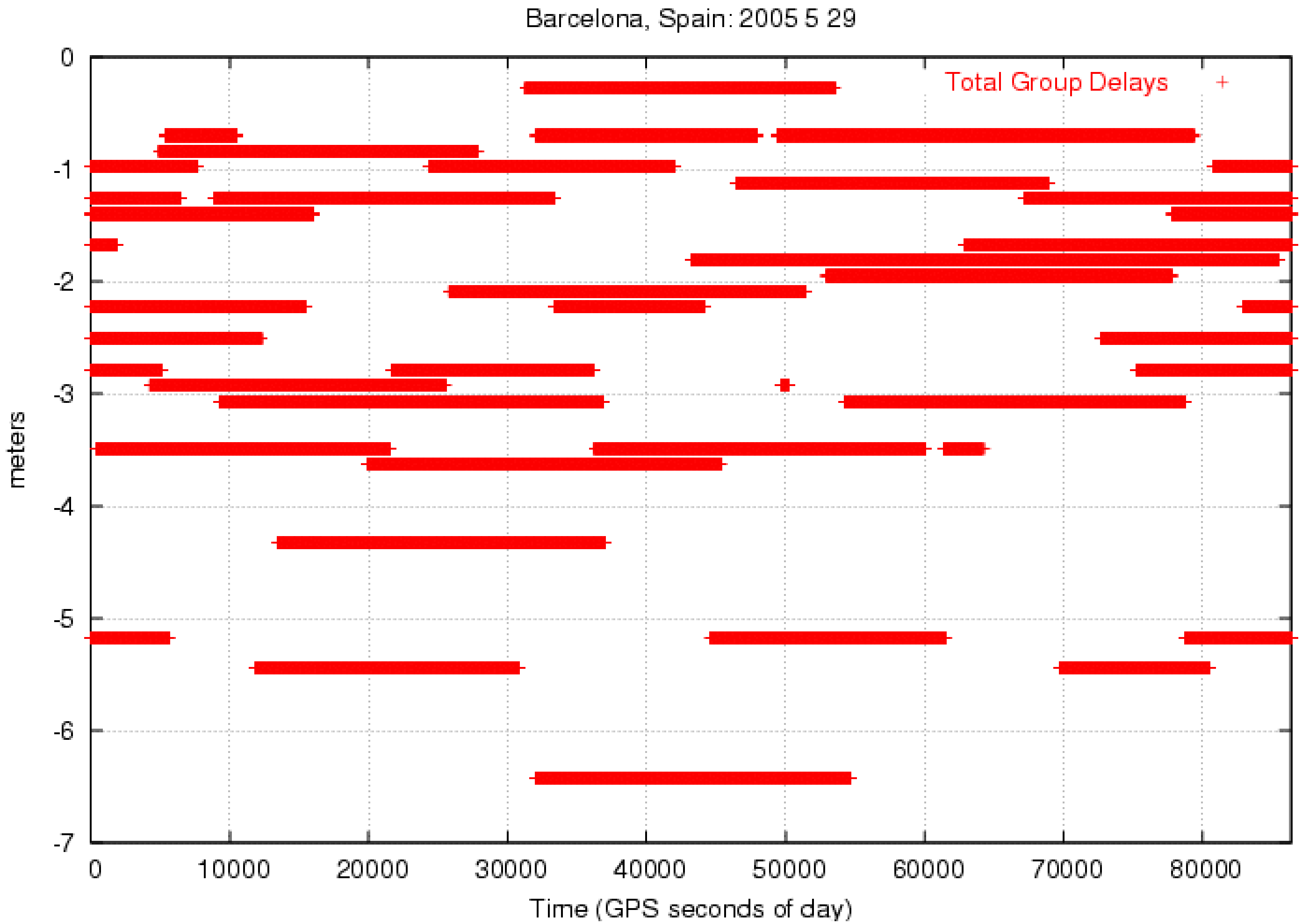
$$K_{2,rec}^{sat} = K_{2,rec} + \frac{f_1^2}{f_2^2} TGD^{sat}$$

- *$K1_{rec}$* may be assumed as zero (including it in receiver clock offset).
- *TGD^{sat}* is transmitted in satellite's navigation message (*Total Group Delay*).

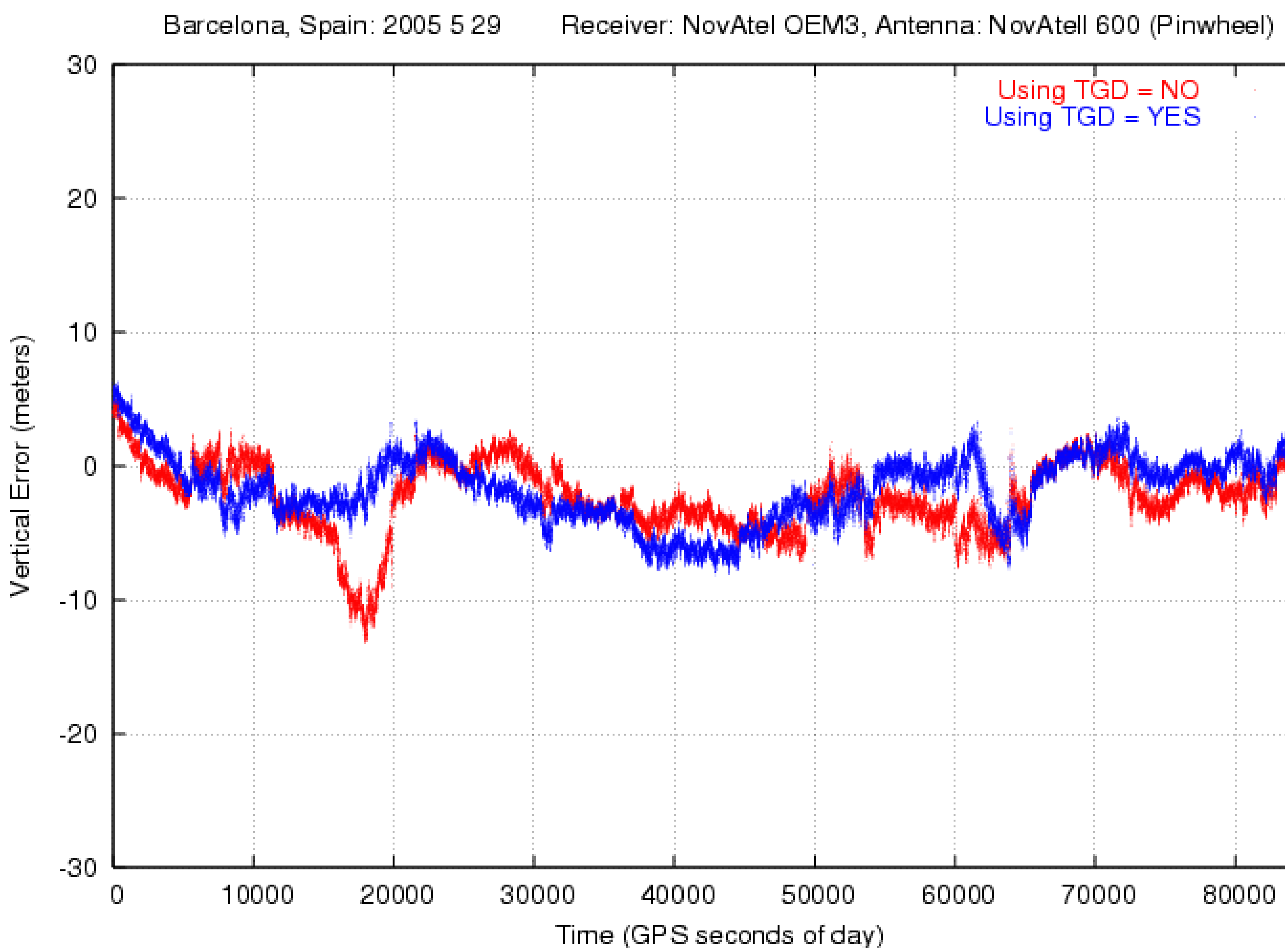
According to ICD GPS-2000, control segment monitors satellite timing, so TGD cancels out when using free-ionosphere combination. That is why we have that particular equation for K_2.

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(\overline{dt}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

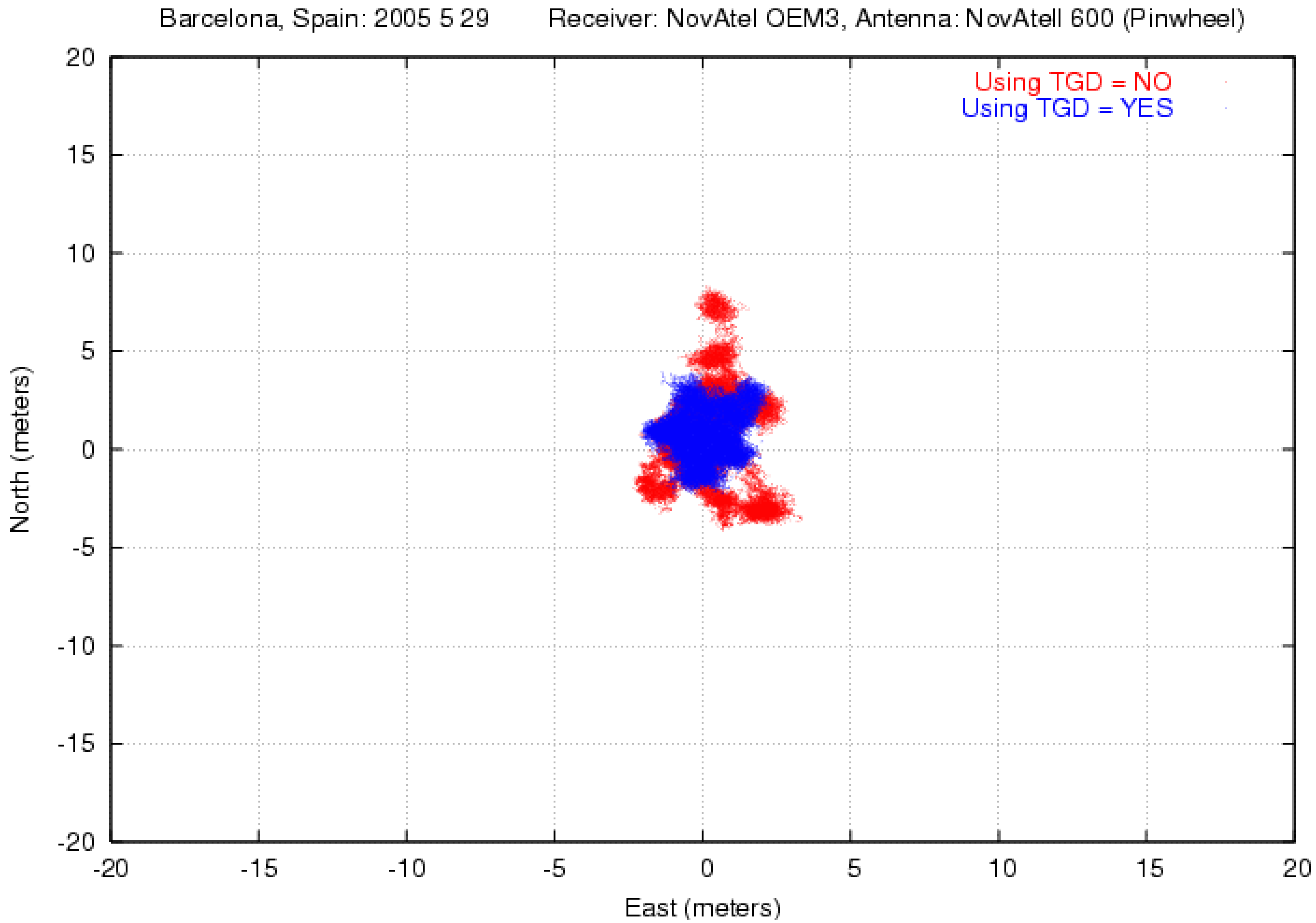
Range variation: Instrumental delays (TGD)



Vertical error comparison



Horizontal error comparison



Measurement noise (thermal noise)

Antispoofing (A/S):
The code **P** is encrypted to **Y**.
→ Only the code **C** at frequency **L1** is available.

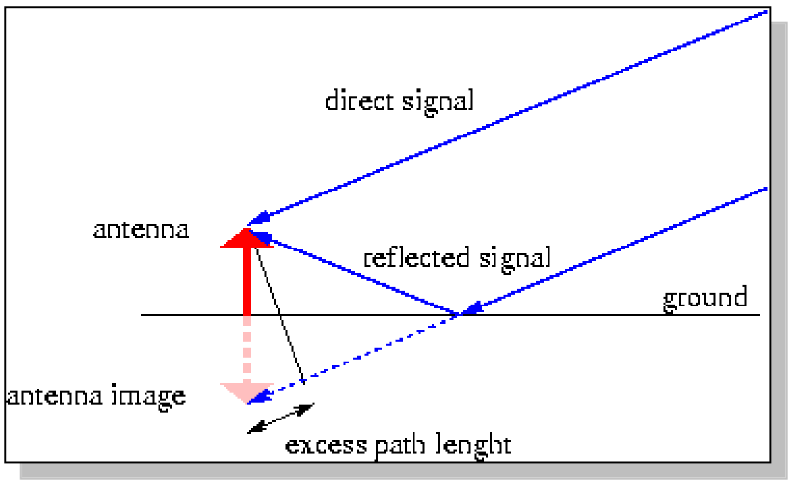
	Wavelength (chip-length)	σ noise (1% of λ) [*]	Main characteristics
Code measurements			
C1	**300 m**	**3 m**	**Unambiguous** but noisier
P1 (Y1): encrypted	30 m	30 cm	
P2 (Y2): encrypted	30 m	30 cm	
Phase measurements			
L1	**19.05 cm**	**2 mm**	**Precise** but ambiguous
L2	**24.45 cm**	**2 mm**	

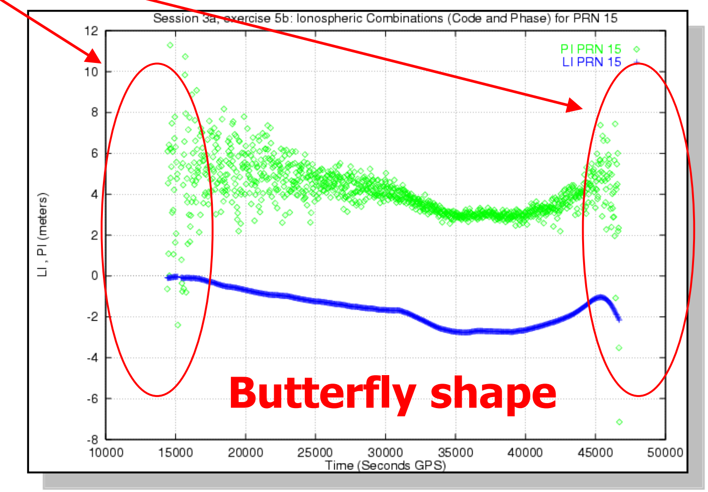
[*] codes may be smoothed with the phases in order to reduce noise (i.e., C1 smoothed with L1 → 50 cm noise)

Multipath

- One or more reflected signals reach the antenna in addition to the direct signal. Reflective objects can be earth surface (ground and water), buildings, trees, hills, etc.

- It affects both code and carrier phase measurements, and it is more important at low elevation angles.

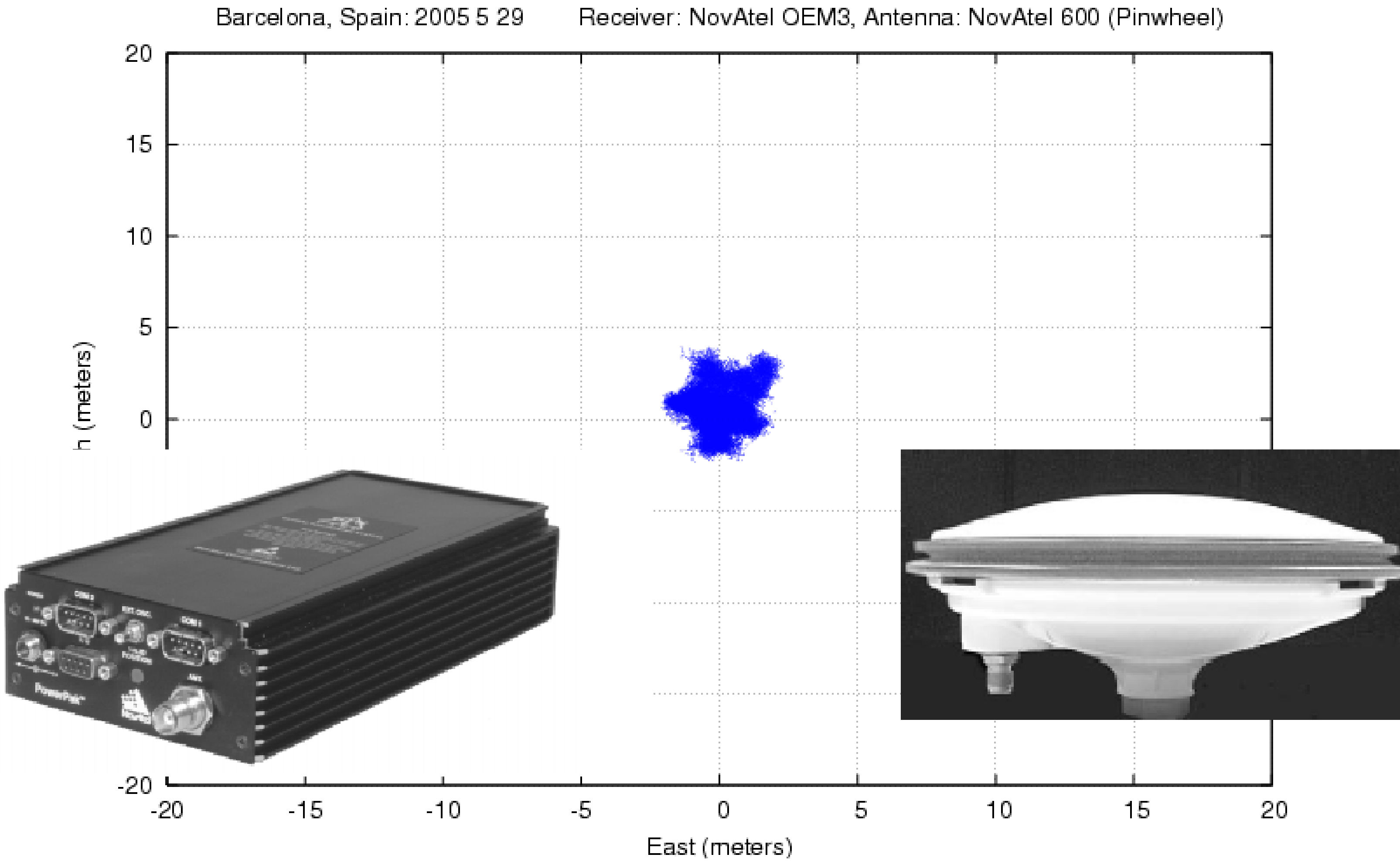




- Code: up to 1.5 chip-length ➔ up to 450m for C1 [theoretically]
  Typically: less than 2-3 m.

- Phase: up to $\lambda/4$ ➔ up to 5 cm for L1 and L2 [theoretically]
  Typically: less than 1 cm

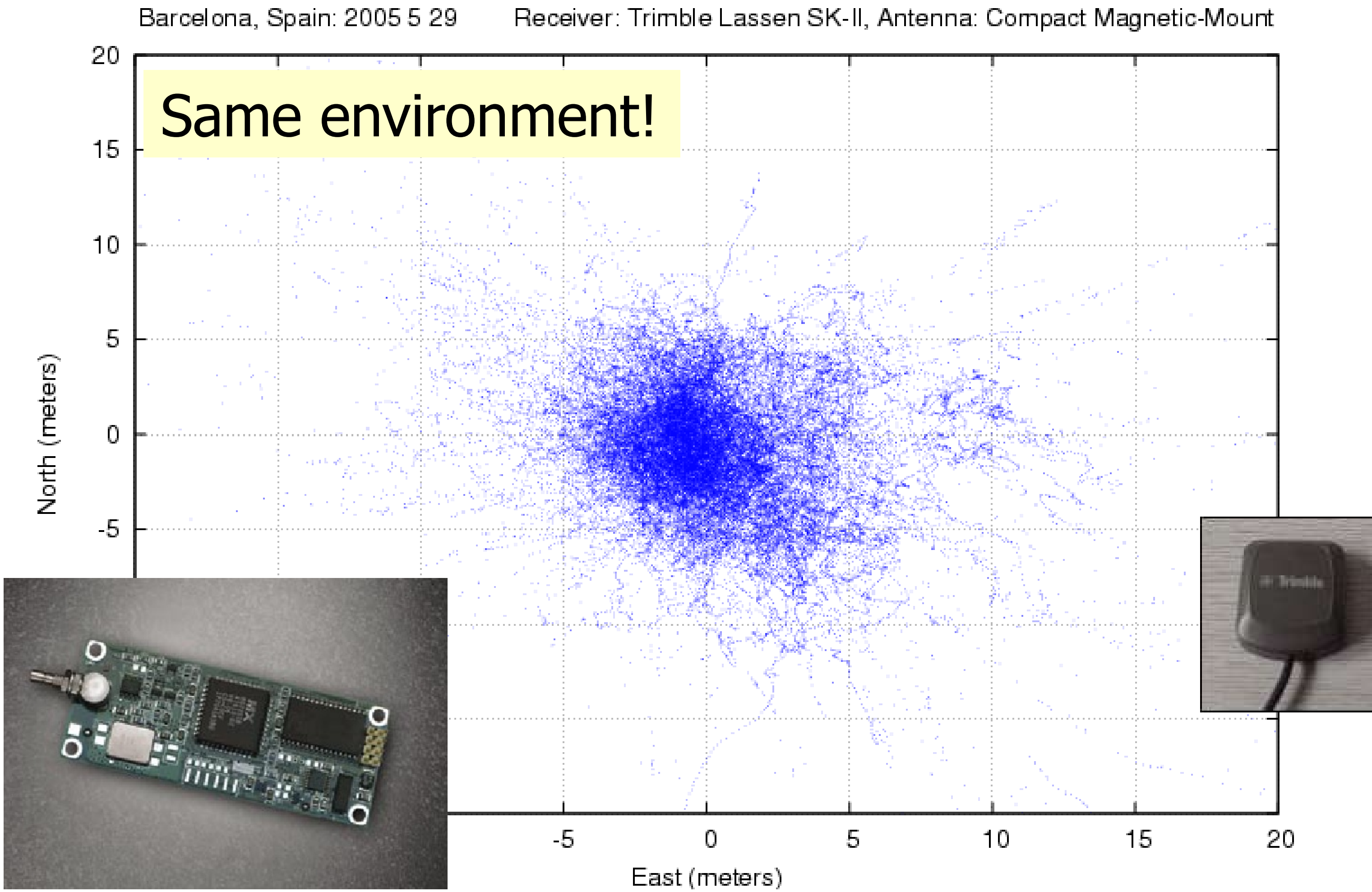
Receiver and multipath noise



GPS standalone (C1 code)

10,000 €

Receiver noise and multipath



GPS standalone (C1 code)

100 €

Contents

Example of Computation of modeled pseudorange

Using data of files **gage2860.98o** and **brdc2860.98n**, compute "by hand" the modeled pseudorange for satellite PRN 14 at t=38230 sec (10h37m10s).

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(\overline{dt}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Follow these steps:

See also exercise 5, Session 5.2 in [RD-2]

1. Select orbital elements closer to 38230
2. Compute satellite clock offset
3. Compute satellite-receiver aprox. geometric range

   3.1 Compute emission time from receiver (reception) time-tags and code pseudorange.

   3.2 Compute satellite coordinates at emission time

   3.3 Compute approximate geometric range.

4. Compute satellite Instrumental delay (TGD):
5. Compute relativistic satllite clock correction
6. Compute tropospheric delay
7. Compute ionospheric delay
8. Compute modeled pseudorange from previous values:

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\bar{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

1. **Selection of orbital elements:** From file **brdc2860.98n**, select the last transmitted navigation message block before instant t=38230 s (10h37m10s).

Transmission time:
979 208818 → 10h 0m 18s

PRN

14 98 10 13 12 0 0 +5.65452501178E-06	+9.09494701773E-13	+0.00000000000E+00	
+1.28000000000E+02	-6.10000000000E+01	+4.38125402624E-09	+8.198042513605E-01
-3.31364572048E-06	+1.09227513894E-03	+5.67547976971E-06	+5.153795101166E+03
+2.16000000000E+05	-6.33299350738E-08	[illegible]952E+00	-3.725290298462E-09
+9.73658001335E-01	+2.74031250000E+02	[illegible]383E+00	-8.081050495434E-09
[illegible]	+1.00000000000E+00	+9.79000000000E+02	+0.00000000000E+00
[illegible]	+0.00000000000E+00	-2.32830643654E-09	+1.28000000000E+02
+2.08818000000E+05	+0.00000000000E+00	+0.00000000000E+00	+0.00000000000E+00

GPS week

GPS sec of week

2. Satellite clock offset computation: From file **brdc2860.98n**, compute satellite clock offset at time t=3830 s for PRN14:

PRN, t_0, a_0, a_1, a_2

```
14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.00000000000E+00
+1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.19804251360 5E-01
-3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.15379510116 6E+03
+2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.72529029846 2E-09
+9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.08105049543 4E-09
-1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.00000000000 0E+00
+3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.28000000000 0E+02
+2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.00000000000 0E+00
```

t = 38230 sec

t_0= 12h 0m 0s= 43200 s

$$\tilde{dt}^{sat} = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 = 5.65\cdot 10^{-6}\,\mathrm{s}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(\tilde{dt}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

3. Satellite-receiver geometric range computation:

Use the following values (4789031, 176612, 4195008) as approximate coordinates.

3.1: Emission time computation from receiver time-tag and code pseudorange:

$$T_{emis} = t_R(T_{recep}) - (C1/c + dt^{sat})$$

Measurement file gage2860.98o	→	Pseudorange *C1* at receiver time-tag t=38230sec: C1= 23585247.703 m
Ephemeris file brdc2860.98n	→	Satellite clock offset at t=38230 sec dt^{sat}= 5.65 10^{-6} sec (see previous results)

Thence, the emission time in GPS system time is:

$$\mathbf{T_{emis}} = 38230 - (23585247.703/c + 5.65\ 10^{-6}) =$$

$$= \mathbf{38229.921\ sec}$$ (where $c=299792458\ m/s$)

Note:

From RINEX measurement file **gage2860.98o**, select the *C1* pseudorange measurement at receiver time-tag for PRN14:

PRN 14

t = 38230 sec= 10h 37m 10s

```
4     L1     L2     C1     P2                                   # / TYPES OF OBSERV
```

```
98 10 13 10 37 10.0000000  0   5G18G14G16G 4G19
   5007753.999           0.000    20143892.105         0.000
   -220595.001           0.000    23585247.703         0.000
   1305085.999           0.000    23146887.826         0.000
   6246118.999           0.000    20798091.711         0.000
 -19853878.999           0.000    22235319.057         0.000
```

Thence:

Measurement file gage2860.98o → Pseudorange C1 at receiver time-tag t=38230sec: C1= 23585247.703 m

3.2: Satellite coordinates at emission time pseudorange:

Temission = 38229.921 sec → **Orbit.f** → X= 11453479.346 Y= 122468524.004 Z= 8245076.145 CTS [emission]

↑ Use the selected ephemeris for PRN14 (from file brdc2860.98n)

The previous coordinates are given in an Earth-Centered-Earth-Fixed reference frame (CTS) at $T_{emission}$= 38229.921s. This reference frame rotates by un amount "$\omega_E \Delta t$" during traveling time $\Delta t = T_{reception} - T_{emission}$.

$$(X^{sat},Y^{sat},Z^{sat})_{CTS[reception]} = R_3(\omega_E\,\Delta t).(X^{sat},Y^{sat},Z^{sat})_{CTS[emission]}$$

$(X^{sat}, Y^{sat}, Z^{sat})_{CTS[reception]} = R_3(\omega_E \Delta t) \cdot (X^{sat}, Y^{sat}, Z^{sat})_{CTS[emission]}$

$$\underbrace{\begin{pmatrix} 11453350.377 \\ 122468589.797 \\ 8245076.145 \end{pmatrix}}_{CTS[reception]} = \begin{pmatrix} \cos(\omega_E \Delta t) & \sin(\omega_E \Delta t) & 0 \\ -\sin(\omega_E \Delta t) & \cos(\omega_E \Delta t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 11453479.346 \\ 122468524.004 \\ 8245076.145 \end{pmatrix}}_{CTS[emission]}$$

$$\omega_E \Delta t = -5.74 \cdot 10^{-6}\, rad. \quad (where \quad \omega_E = 7.2921151467 \cdot 10^{-5}\, rad/\sec)$$

$$\Delta t = -\frac{\rho_{0,rec}^{sat}}{c} = -0.079 \sec$$

$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2} \approx 23616673.3m$$

$$(x, y, z)^{satellite} \approx (11453479,\ 122468524,\ 8245076)$$

$$(x_0, y_0, z_0)_{receiver} \approx (4789031,\ 176612,\ 4195008)$$

An approximate value is enough to compute Δt.

Note: Both satellite and receiver coordinates must be given in the same reference system!

➔the CTS[reception] will be used to build navigation equations.

3.2: Geometric range computation

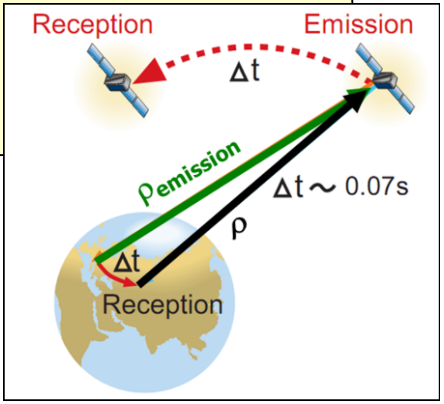
The geometric range between satellite coordinates at emission time and the "approximate position of the receiver" at reception time *(both coordinates given in the same reference system [for instance the CTS system at reception time])* is computed by:

$$\rho_{0,receiver}^{satellite} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2} = 23616699.124m$$

$$(x,y,z)^{satellite} = (11453350.2771,\ 22468589.7975,\ 8245076.1448)_{CTS[reception]}$$

$$(x_0, y_0, z_0)_{receiver} = (4789031,\ \ 176612,\ 4195008)_{CTS[reception]}$$

"Approximate" receiver coordinates at reception time.



$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\bar{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

4. Time Group Delay (TGD) or Satellite Instrumental delay.

➔ From file brdc2860.98n, compute the TGD for PRN14:

PRN

TGD (in sec)

```
14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
+1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.19804251360E-01
-3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.15379510116E+03
+2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.72529029846E-09
+9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.08105049543E-09
-1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.000000000000E+00
+3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.28000000000E+02
+2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.000000000000E+00
```

TGD= -2.32830643654E-09 * c= -0.69801 m

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(\bar{dt}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

5. Relativistic clock correction:

PRN (14)   e   sqrt(a)

```
14 98 10 13 12 0 0 +5.6545251178E-06 +9.09494701773E-13 +0.000000000000E+00
+1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.198042513605E-01
-3.3136457204 8E-06 +1.09227513894E-03 +5.67547976971E-06 +5.15379510116 6E+03
+2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.725290298462E-09
+9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.081050495434E-09
-1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.000000000000E+00
+3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.280000000000E+02
+2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.000000000000E+00
```

Temission = 38229.921 s → *Orbit.f* → *E = 0.095 rad (eccentric anomaly)*

$$\Delta rel^{sat} = -2\frac{\sqrt{\mu a}}{c^2}\, e \sin(E) = -2.3\cdot 10^{-10}\,\mathrm{s}$$

$\mu = 3.986005\cdot 10^{14} \quad m^3 s^{-2}$

$c = 299792458 \quad m\, s^{-1}$

$$C1_{rec}^{sat}[\mathrm{modelled}] = \rho_{0,rec}^{sat} - c\left(d\bar{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

6. Tropospheric correction

$$Trop_{rec}^{sat} = \left(d_{dry} + d_{wet}\right) m(elev) = 6.76m$$

$$d_{dry} = 2.3\,e^{-0.116 \cdot 10^{-3} H} = 2.3m$$

$$d_{wet} = 0.1m$$

$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

See klob.f

$$elev = 20.57 \frac{\pi}{180} = 0.359 rad$$

$$H = 160m \quad (height\ \ over\ the\ ellipsoid)$$

(x,y,z)$_{rec}$ ➔[car2geo] ➔ (Lon, Lat, H)$_{rec}$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\bar{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

7. Ionospheric correction

(time, r_{sta}, r^{sat}, $\alpha0,\alpha1,\alpha2,\alpha3,\beta0,\beta1,\beta2,\beta3$) → [Klob] → Iono=10.26m

```
 2              NAVIGATION DATA     GPS                 RINEX VERSION/ TYPE
XPRINT v1.1         gAGE               00/06/04 17:36:23    PGM / RUN BY / DATE
gAGE BROADCAST EPHEMERIS FILE                               COMMENT
   +1.9558E-08 +0.0000E+00 -1.1921E-07 +0.0000E+00          ION ALPHA
   +1.2288E+05 -1.6384E+04 -2.6214E+05 +1.9661E+05          ION BETA
   -8.381903171539E-09-1.421085471520E-14   405504      979 DELTA_UTC: A0,A1,T,W
    12                                                      LEAP SECONDS
                                                            END OF HEADER
```

$$t = 38230\,\text{sec}$$

$$(x,y,z)^{satellite} = (11453350.2771,\ 22468589.7975,\ 8245076.1448)_{CTS[reception]}$$

$$(x_0,y_0,z_0)_{receiver} = (4789031,\ 176612,\ 4195008)_{CTS[reception]}$$

Approximate values for receiver or satellite coordinates are enough

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

7. Compute the modeled pseudorange.

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

$$\rho_{0,rec}^{sat} = 23616699.124\,\text{m}$$

$$c\,d\overline{t}^{sat} = 5.65\cdot 10^{-6}\,c = 1693.828\,\text{m}$$

$$c\,\Delta rel^{sat} = -2.33\cdot 10^{-10}\,c = -0.071\,\text{m}$$

$$Trop_{rec}^{sat} = 6.760\,\text{m}$$

$$Ion_{1rec}^{sat} = 10.260\,\text{m}$$

$$TGD^{sat} = -0.698\,\text{m}$$

$$\rightarrow\; C1_{rec}^{sat}[\text{modelled}] = 23615021.689\text{m}$$

Prefit residual:

Is the difference between measured and modeled pseudorange

$$\text{Pref}_{rec}^{sat} = C1_{rec}^{sat} - C1[\text{mod}]_{rec}^{sat} = \rho_{rec}^{sat} - \rho_{0,rec}^{sat} + c\,dt_{rec} + K_{1rec} + \varepsilon$$

In the previous example (PRN14 at t = 38230 s):

Pref= 23585247.703 - 23615021.689 = -29773.986 m

Previously calculated

From measurement file

References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga –Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

[RD-5] Rovira-Garcia A, Juan J, Sanz J, Gonzalez-Casado G (2015) A Worldwide Ionospheric Model for Fast Precise Point Positioning. Geoscience and Remote Sensing, IEEE Transactions on 53(8):4596{4604, DOI 10.1109/TGRS.2015.2402598, URL http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=7053952

Thank you

gAGE

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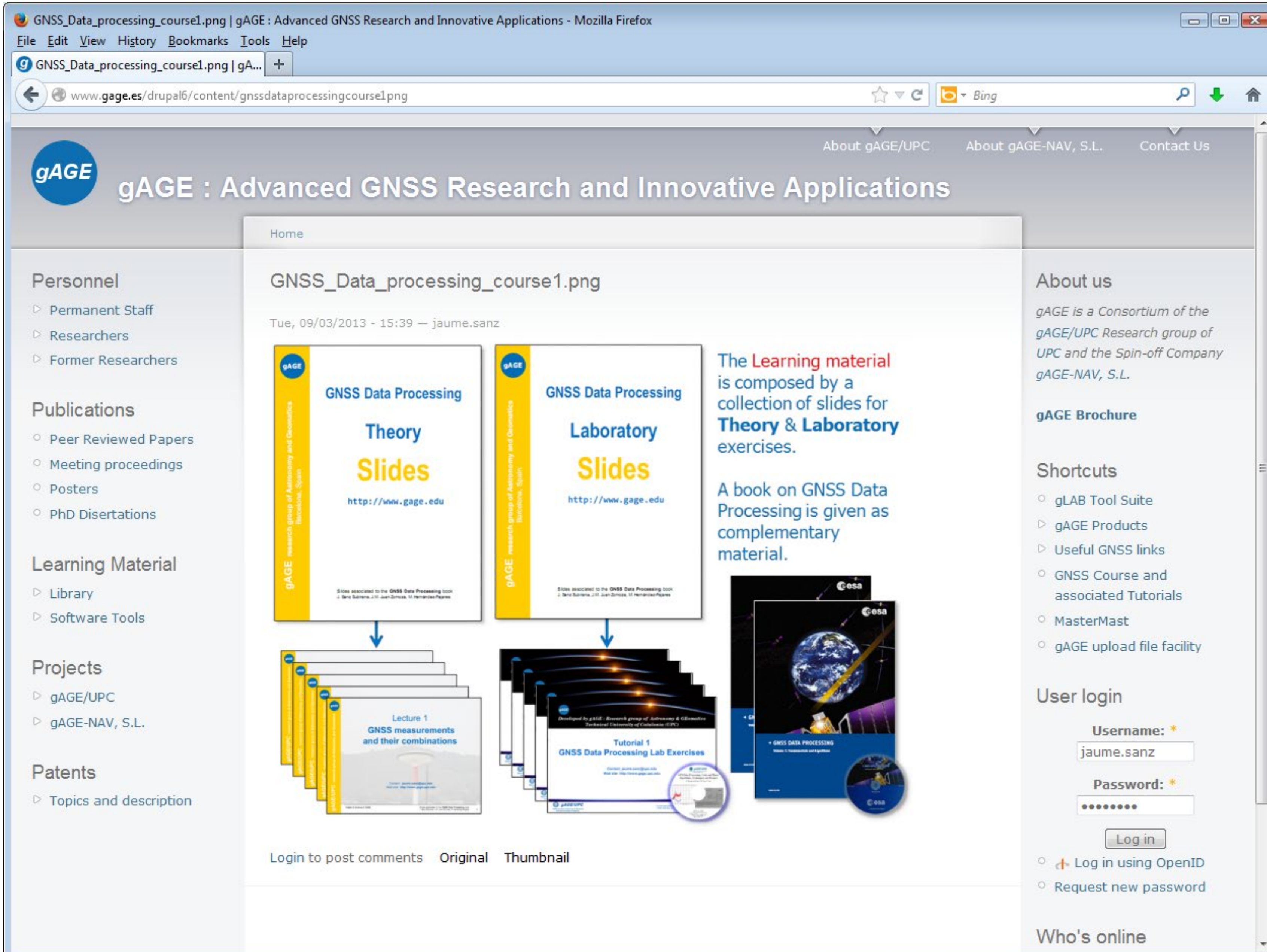

GNSS Data Processing, Vol. 1: Fundamentals and Algorithms.
GNSS Data Processing, Vol. 2: Laboratory exercises.